

An Empirical Study of the Relation between Stock Market Returns and Volatility in the BRVM

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Abstract

This paper studies the relationship between expected stock market returns and volatility in the regional stock market of the West African Economic and Monetary Union called the BRVM. Using weekly returns over the period 4 January 1999 to 29 July 2005 and, an EGARCH-in-Mean model assuming normally distributed and Student's *t* distribution for error terms, the study reveals that: 1) expected stock return has a positive but not statistically significant relationship with expected volatility. 2) volatility is higher during market booms than when market declines.

Keywords: Regional stock market, BRVM, WAEMU, EGARCH-M, Risk-returns tradeoff.

JEL Classification Codes: G11; G12; G15

1. Introduction

The relationships between expected returns and expected volatility have been extensively examined over the past years. Theory generally predicts a positive relation between expected stock returns and volatility if investors are risk averse¹. That is equity premium provides more compensation for risk when volatility is relatively high. In other words, investors require a larger expected return from a security that is riskier. Yet, empirical studies that attempt to test this important relation yield mixed results.

Estimates of the risk-return relation exploiting the GARCH-M framework range from positive (French et al., 1987; Chou, 1988; Baillie and DeGennaro (1990), Campbell and Hentschel, 1993; Scruggs, 1998; and Bansal and Lundblad, 2002) to negative (Nelson, 1991; Glosten et al., 1993). On the one hand, French et al. (1987) examine daily and monthly returns on the NYSE stock index for the period from January 1928 to December 1984 and find evidence that expected market risk premium is positively related to predictable volatility of stock returns. Using the same source of data, but for a different period, Chou (1988) supported French et al.'s finding about the positive relation between the predictable components of stock returns and volatility. Chou studied weekly data for the period July 1962 to December 1985. Baillie and DeGennaro study similar data to French et al. and Chou, and reached the same conclusion. They study both daily data for the period 1 January 1970 to 22 December 1987, and monthly data for the period February 1928 to December 1984. On the other hand, Glosten et

¹ Backus and Gregory (1993) argue that the theoretical relation between market risk premium and conditional market variance is not necessarily a positive linear function. In general, the function depends on the preferences of the representative agent and the stochastic nature of the economy.

al. (1993) use data on the NYSE over April 1851 to December 1989, and find negative relationship between expected stock market return and volatility.

Alternatives to the GARCH-M framework have been used but also yielded mixed results. Using an instrumental variables specification for conditional moments, Campbell (1987) finds negative risk-return tradeoff whereas Harvey (1991) finds a positive relationship, and Whitelaw (1994) find mixed evidence on the expected return volatility tradeoff. Turner et al. (1989) use a two-stage Markov model and find that the relationship between expected stock returns and volatility range from positive to negative. Using non-parametric techniques, Pagan and Hong (1991) find a weak negative relationship, but Harrison and Zhang (1999) find that the relationship is significantly positive at longer horizons. For a specification that facilitates regime-switching, Whitelaw (2000) documents a negative unconditional link between the mean and variance of the market portfolio.

Given the conflicting results cited above, it is primary an empirical question as to whether the conditional first and second moments of equity returns are positively related.

The purpose of this paper is to contribute to this literature by examining the relation between expected stock market returns and expected volatility in the BRVM over the period from 4 January 1999 to 29 July 2005. The contribution of this paper is threefold. First, it uses data on a frontier market and tests for the risk-returns tradeoff in the BRVM for the first time. Second, it contributes to the literature on this important relation by showing that the risk-returns tradeoff in the BRVM is conform to those found in mature markets. Third, it shows that the asymmetric coefficient is positive in the BRVM contrary to those found in mature markets.

The rest of the paper is organised as follows: section 2 gives an overview of the BRVM. Section 3 describes the data. Section 4 exposes the econometric methodology. Section 5 concludes the paper.

2. Overview of the BRVM

The Bourse Régionale des Valeurs Mobilières hereafter BRVM is a private corporation set up on 18 December 1996 but began operations in September 16, 1998. Its mission is to organize the securities market; disseminate market information; and promote the market. It is the Regional Financial Exchange of Benin, Burkina-Faso, Guinea Bissau, Côte d'Ivoire, Mali, Niger, Senegal, and Togo which form the West African Economic and Monetary Union.

The BRVM is a centralized spot exchange driven by orders, that is, the price of a security is fixed by matching bid and ask orders collected before the quotation. At inception, it has three trading sessions a week, on Mondays, Wednesdays and Fridays. Prices are set at a fixing session that will gradually become an ongoing process. A second fixing is done before the end of the trading session such that securities "unlisted" and/or "reserved" during the exchange's first fixing can eventually be traded.

The BRVM has an electronic system and a satellite network that allows brokerage firms to send orders from the various WAEMU member states to the central site located in Abidjan, Cote d'Ivoire. When operations began, the BRVM had two sections for stocks and one section for bonds: the first section for stocks is reserved for companies that can justify at least five certified annual accounts, a market capitalization of over 500,000,000 CFA francs² and distributed public shares of at least 20%. The second section for stocks can be accessed by mid-sized companies with market capitalization of at least 200,000,000 CFA francs and two years of certified accounts, and a commitment to distributing at least 20% of their capital to the public within two years, or 15% in the event of a share capital increase; the bond section can be accessed via bond loans of which the total number of shares issued is higher than 25,000 and the face value of the share is equal to at least 500,000,000 CFA francs.

² The CFA franc, which stands for Communauté Financière Africaine is the common currency shared by WAEMU 8 member States. 1 Euro = 655.957 CFA Franc at a fixed rate.

Two BRVM market indexes represent the activities of stock market shares: the BRVM Composite comprises all securities listed on the exchange, and the BRVM 10 comprises the ten most active companies on the exchange (see Table 1 below).

Table 1: Selected figures of the BRVM

Capitalization	31 Dec. 2002	31 Dec. 2003	31 Dec. 2004	29 July 2005
BRVM 10	465 634 264 590	617 337 595 495	607 239 551 350	766 467 440 025
BRVM Composite	832 398 094 700	858 140 223 580	1005 047 884 085	1094 198 936 835
Number of firms				
BRVM 10	10	10	10	10
BRVM Composite	39	39	39	39
Some measures				
Volume traded of stocks	4823	2994	2031	1033
Total value traded (CFAfr)	171 254 520	72 584 325	35 861 220	47 493 520
# of transations	30	85	28	25
# of transations	9	15	8	8

Notes: 1 Euro = 655.957 CFA francs. The CFA franc is the currency unit of the West African Economic and Monetary Union (WAEMU) 8 member States.

Source: Official Newsletter of the BRVM.

3. Data Description

The data set used in this study is weekly closing prices on the BRVM 10 index obtained from the Official Newsletter of the Regional Stock Market (BRVM). The study period ranges from 4 January 1999 to 29 July 2005. The choice of the BRVM 10 over the BRVM Composite is motivated by two reasons: first, it is composed of the ten most actively traded stocks in the BRVM and second, it accounts for about 70% of the total market capitalization of the BRVM as shown in Table 1 above. I compute the weekly stock market returns, R_t , as follows:

$$R_t = 100 * (\ln P_t / P_{t-1}) \quad (1)$$

where P_t is the value of the BRVM 10 price index for the period t , and t represents time in weeks. P_{t-1} is BRVM 10 index price for period $t-1$; $\ln(\cdot)$ is the logarithm operator. All returns are expressed in local currencies and are not adjusted for dividends. Table 2 below reports summary statistics of weekly stock market returns for the BRVM equity market.

Table 2: Summary statistics for weekly return

Series	Mean	S.D	S	K	J.B.	Q(6)	Q(20)	Q ² (6)	Q ² (20)
BRVM 10	0.072	1.636	0.930	7.635	354.350 (0.000)	69.376 (0.000)	89.496 (0.000)	61.505 (0.000)	81.140 (0.000)

Notes: Mean, S.D., S, K, J.B. are the sample mean, standard deviation, skewness, and the kurtosis respectively. $Q(\cdot)$ and $Q^2(\cdot)$ are the Ljung-Box Q-statistics and the squared Ljung-Box Q-statistics respectively. P-values are in parentheses.

The mean and the standard deviation are 0.072 and 1.636 respectively. The skewness statistic of 0.930 shows that the distribution is positively skewed relative to the normal distribution (0 for the normal distribution). This is an indication of a non symmetric series. The kurtosis is very much larger than 3, the kurtosis for a normal distribution. This suggests that for the BRVM, large market surprises of either sign are more likely to be observed, at least unconditionally. The Ljung-Box test statistics $Q(\cdot)$ and $Q^2(\cdot)$ provide tests for the absence of autocorrelation and homoscedasticity, respectively. The significance values of Q -statistics indicate significant serial correlation in the mean return series. This suggests that the inclusion of a lag dependent variable in the mean equation is appropriate. Strong autocorrelation is also detected in the squared mean returns as shown by the values of the $Q^2(\cdot)$. It

results in volatility clustering in the distribution of stock market returns. In addition, the Jarque-Bera normality test rejects the hypothesis of normality.

4. Econometric Methodology

4.1. Relation between expected return and expected volatility

In order to examine the relation between expected returns and expected volatility, I exploit the the GARCH-in-Mean technology (Engle et al. 1987). The motivation for this choice stems from the fact that the expected return on an asset is proportional to the expected risk of that asset. I assume that the mean component in the GARCH (1, 1)-in-Mean framework describes the expected returns-volatility tradeoff for the equity market returns as follows:

$$R_t = b_0 + b_1 R_{t-1} + \delta \sigma_t^2 + \varepsilon_t \quad (2)$$

where R_t represents stock market returns at time t , the lag order of the autoregressive process for equation (2) is determined by the Schwartz (1978) criteria. The optimum lag is one. R_{t-1} is the returns at time $t-1$ accounting for autocorrelation, b_0 is comparable to the risk-free rate in the Capital Asset Pricing Model, $\delta \sigma_t^2$ is the market risk premium for expected volatility, ε_t is the disturbance terms with mean zero and conditional variance σ_t^2 . The expected volatility is approximated by σ_t^2 , the conditional variance of R_t such that:

$$\sigma_t^2 = \text{var}(R_t / \psi_{t-1}) \quad (3)$$

Where R_t is defined as above, ψ_{t-1} is the information set up to time $t-1$ and, $\text{var}(\cdot)$ is the variance operator.

The volatility measure defined by the conditional variance above is in an expectation formulation. If the forecasts of this variance can be used to predict expected returns, then we should expect the coefficient δ in equation (1) to be positive and significant for a risk averse investor. In other words, if investors are rewarded for their exposure to risk, then we should expect a positive relation between conditional expected returns and conditional variance. This supposes that markets are fully segmented, that is investors do not diversify their portfolio internationally. Therefore, they should be rewarded for their exposure to country specific risk. The coefficient δ in equation (1) that links first and second moment of returns can be interpreted as the price of domestic market risk.

Estimating and testing the risk returns tradeoff described in equation (1) requires an empirical model for the conditional volatility. My choice of models is motivated by the empirical literature on market volatility. I assume that the conditional variance follows an Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) process (Nelson, 1991). The GARCH (Bollerslev, 1986) family of models assumes that the market conditions its expectation of market variance on both past conditional market variance and past market innovations. The EGARCH model, a refinement of the GARCH model imposes a nonnegativity constraint on market variance, and allows for conditional variance to respond asymmetrically to return innovations of different signs. I specify the model as follows:

$$\ln \sigma_t^2 = w + \beta \ln \sigma_{t-1}^2 + \alpha \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (4)$$

where w, β, α, γ are constant parameters, $\ln(\sigma_t^2)$ is the one-period ahead volatility forecast. This implies that the leverage effect is exponential rather than quadratic and forecast of conditional variance are guaranteed to be nonnegative, w is the mean level, β is the persistence parameter, $\ln(\sigma_{t-1}^2)$ is the past period variance. Unlike the GARCH model, the EGARCH model allows for leverage effect. If γ is negative, leverage effect exists. That is an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude (Black, 1976;

Christie, 1982). If α is positive, then the conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller).

Equations (2) and (4) are jointly estimated after specifying the assumptions about the distribution of error terms. I consider two distributions for error terms: the normal distribution and the Student's t distribution. The choice of the former is dictated by the fact that estimation traditionally assumes normally distributed error terms. This case will serve as a benchmark for comparison. The fact that excess skewness and kurtosis displayed by the residuals of conditional heteroskedastic models will be reduced if a more appropriate distribution is used, justifies the use of Student's t-distribution for error terms. Equations (2) and (4) can be summarized in the following two models:

Model 1:

$$R_t = b_0 + b_1 R_{t-1} + \delta \sigma_t^2 + \varepsilon_t \tag{5}$$

$$\varepsilon_t / \psi_{t-1} \rightarrow N(0, \sigma_t) \tag{6}$$

$$\ln \sigma_t^2 = w + \beta \ln \sigma_{t-1}^2 + \alpha \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{7}$$

Model 2:

$$R_t = b_0 + b_1 R_{t-1} + \delta \sigma_t^2 + \varepsilon_t \tag{8}$$

$$\varepsilon_t / \psi_{t-1} \rightarrow t(0, \sigma_t^2, \nu_t), \nu_t \text{ is the degree of freedom} \tag{9}$$

$$\ln \sigma_t^2 = w + \beta \ln \sigma_{t-1}^2 + \alpha \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{10}$$

4.2. Empirical results

Table 3: Risk>Returns estimates

	b_0	$b1$	δ	ω	β	α	γ	LF
Model 1	0.008 (0.085)	0.293* (4.761)	0.047 (0.905)	-0.268* (-2.821)	0.718* (8.235)	0.594* (6.080)	0.251* (2.353)	-568.791
Model 2	-0.005 (-0.061)	0.278* (4.992)	0.011 (0.338)	-0.270* (-2.002)	0.720* (8.491)	0.757* (3.555)	0.291* (2.372)	-542.753

Notes: Model 1 is the AR(1)-EGARCH(1,1)-M with normally distributed error terms; model 2 is the AR(1)-EGARCH(1,1)-M with Student's t distribution for error terms. The degree of freedom's coefficient of Model 2, ν_t is 3.019 with a t-statistics of 4.283. L.F. is the maximum value of the log likelihood function; t-statistics are in parentheses. * indicates statistical significance at the 5% level.

The results of estimating models 1 and 2 show that the differences in maximum log-likelihood between the two models are very small. Model 2 produces a marginally higher log-likelihood value. However, in all cases, the differences are very insignificant at the 5% level.

In the mean equations (5) and (8), the coefficient δ of the σ_t^2 term turns out to be positive but statistically insignificant at the 5% level. This result implies that stock returns are not affected by volatility trends. In other words, conditional variance lacks predictive power for stock returns. This result is similar to those reached by French et al. (1987), Baillie and DeGennaro (1990) and, Chan et al. (1992).

However, the literature has not reached yet a consensus on this important relation. According to finance theory, conditional expected returns should be positively and statistically significant in relation to conditional variance (Campbell and Hentschell, 1992). The present study suggests that investors are not rewarded for the risk they take on the regional stock exchange. If they were, the coefficient δ should have been positive and statistically significant.

In terms of the conditional volatility, the persistence parameters β in equations (7) and (10) are 0.718 and 0.720 respectively. This suggests that the degree of persistence is high and very close to one. In other words, once volatility increases, it is likely to remain high over several periods. The positive and statistically significant coefficient α in both models confirms the presence of volatility clustering.

Conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). The positive and statistically significant coefficient γ in both models implies the presence of asymmetry; that is volatility is higher during market booms than when market declines. This result is at odd with those found in the U.S. equity market (Pagan and Schwert, 1990; Nelson, 1991). An explanation of this contradictory result is that investors believe that market booms are not supported by economic fundamentals and market returns behave as speculative bubbles.

4.3. Diagnostic checks

Table 4: Diagnostic checks

	Raw series	Model 1	Model 2
Mean	-0.072	0.038	0.013
Standard deviation	1.636	0.999	0.892
Skewness	0.930	0.044	0.020
Kurtosis	7.635	5.881	6.060
Q(6)	69.376 (0.000)	6.342 (0.386)	7.011 (0.320)
Q(20)	89.496 (0.000)	21.391 (0.374)	21.921 (0.345)
Q ² (6)	61.505 (0.000)	3.929 (0.688)	3.978 (0.680)
Q ² (20)	81.140 (0.000)	13.091 (0.873)	12.480 (0.899)
J.B.	354.350 (0.000)	117.689 (0.000)	122.976 (0.000)

Notes: Table 4 shows the summary statistics for the raw returns and the standardized residuals for Model 1 and Model 2. Q(.) is the Ljung-Box Q-statistics for the absence of autocorrelation and Q²(.) is the squared Ljung-Box Q-statistics for the absence of heteroskedasticity. P-values are in parentheses and, J.B. is the Jaraue Berra test for normality.

Table 4 above reports the summary statistics for the raw returns and the standardized residuals for models 1 and 2. The purpose of this diagnostic check is to test whether the models are correctly specified.

The kurtosis is now 5.881 for model 1 and 6.060 for model 2, which is quite an improvement from the raw series (7.634). Furthermore, the skewness is close to zero for both models. The Q-statistics for the absence of autocorrelation in the standardized residuals have p-values ranging from 0.345 to 0.386 while they were 0.000 in the original series. This confirms the fact that returns have no remaining ARCH effects. The p-values of the Q²-statistics for the absence of heteroscedasticity range from 0.688 to 0.899 relative to 0.000 in the original series. All these tests suggest that the models are fairly specified. They can therefore be used for forecasting purposes.

5. Conclusion

This study has analysed the relationship between stock market returns and volatility in the regional stock market of the West African Economic and Monetary Union called the Bourse Régionale des Valeurs Mobilières (hereafter BRVM).

Using weekly data on stock prices from the Official Newsletter of the BRVM over the period 4 January 1999 through 29 July 2005, the study tests for the risk-returns tradeoff within an EGARCH-in-Mean framework. The study reveals that coefficients linking conditional market returns to conditional volatility are positive but statistically insignificant. This result is conform to results found in mature markets but is at odd with the positive and statistically significant risk-return tradeoff prescribed by finance theory. The result also shows that volatility is persistent but contrary to the EGARCH model of Nelson (1991), there is no leverage effect.

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