International Research Journal of Finance and Economics
ISSN 1450-2887 Issue 14 (2008)

http://www.eurojournals.com/finance.htm

On the Distribution of Returns & Memory Effects in Indian Capital Markets

J.P. Singh
Professor, Department of Management Studies, Indian Institute of Technology
Roorkee, Roorkee 247 667 (Uttaranchal), India
E-mail: jatinder_pal2000@yahoo.com; Jatinfdm@iitr.ernet.in
Tel: +91 1332 85783; +91 1332 85565

S. Prabakaran
Research Scholar, Department of Management Studies
Indian Institute of Technology, Roorkee, Roorkee 247 667 (Uttaranchal), India

Abstract

This paper examines the various features of the logarithmic return spectrum of the Indian stock markets, performing thereon the various statistical tests for the normality of data. It also investigates the possible existence of dependencies and memory effects in the return processes. In particular, it performs rescaled range analysis and carries on to compute the Hurst's exponent. The results throw up several intriguing issues of relevance to portfolio managers, stock market players and analysts and academicians.

1. Introduction

There exist two traditional approaches to the modelling of a dynamical system. In the first approach, the dynamical deterministic equations of motion are obtained from first principles as differential/difference equations that are integrated forward in time and solved as an initial value problem. This methodology, although strongly preferred due to its exactness, is sometimes impracticable, particularly when we are analysing the dynamics of many particle systems with complicated interactions among the constituents. In such cases, either the number of degrees of freedom become so large as to make the first-principles model intractable or the initial conditions pertaining to each degree of freedom become inaccessible. Attempts are, then, made to model the dynamics as a random process with stochastic, though linear, laws of motion. There was believed to be no region of overlap between these two well-defined approaches.

The modus operandi for studies on stock market phenomena was no different and one could go to the extent of saying that the Efficient Market Hypothesis [1-2] was formulated with one primary objective – to create a scenario that would justify the use of stochastic calculus [3] for the modeling of capital markets.

The cardinal maxim of the Efficient Market Hypothesis is the existence of a market where all assets are fairly priced according to the information available with neither the buyers nor sellers enjoying any advantage. Market prices are supposed to incorporate all publicly accessible information, both fundamental and price history. It is, further, postulated that prices move only as sequel to new information entering the market. The presence of large number of investors ensures that all prices are fair. Memory effects, if any at all, are extremely short ranging and dissipate rapidly. Feedback effects on prices are, thus, assumed to be marginal. The investor community is considered rational as benchmarked by the traditional concepts of risk and return.

An immediate corollary to the Efficient Market Hypothesis is the independence of single period returns, so that they may be modeled as a random walk and the defining probability distribution, in the limit of the number of observations being large, would be the normal distribution.

While several adaptations of the Random Walk Efficient Market Hypothesis have been postulated with the corresponding variants of the normal distribution and martingale and submartingale approaches [4-7] being used to model stock prices or returns, the variance continues to be the cardinal measure of risk, as defined by the volatility of stock prices.

Ever since the studies of Fama in 1964-65, evidence has been accumulating against the validity of the Efficient Market Hypothesis – the existence of negatively skewed observations and fat tails and distortion around the mean values are but a few [1,4, 8-11].

One of the most exhaustive sets of studies on stock market data in varying dimensions has been reported in [12-16]. In [16], a phenomenological study was conducted of stock price fluctuations of individual companies using data from two different databases covering three major US stock markets. The probability distributions of returns over varying timescales ranging from 5 min. to 4 years were examined. It was observed that for timescales from 5 minutes upto 16 days the tails of the distributions were well described by a power law decay. For larger timescales results consistent with a gradual convergence to Gaussian behaviour was observed. In another study [12] the probability distributions of the returns on the S & P 500 were computed over varying timescales. It was, again, seen that the distributions were consistent with an asymptotic power law behaviour with a slow convergence to Gaussian behaviour. Similar findings were obtained on the analysis of the NIKKEI and the Hang – Sang indices [12].

Furthermore, the access to enhanced computing power during the last decade has enabled analysts to try refined methods like the phase space reconstruction methods for determining the Lyapunov Exponents [17] of stock market price data, besides doing Rescaled Analysis [18] etc. Results of these studies have unambiguously established the existence of significant nonlinearities and chaotic behavior in these time series [19-22].

This paper examines the various features of the logarithmic return spectrum of the Indian stock markets, performing thereon the various statistical tests for the normality of data. It also investigates the possible existence of dependencies and memory effects in the return processes. In particular, it performs rescaled range analysis and carries on to compute the Hurst's exponent. The results throw up several intriguing issues of relevance to portfolio managers, stock market players and analysts and academicians.

2. Testing & Evidence on the Normality of Stock Market Returns in India

As mentioned above, the pivotal fallout of the Efficient Market Hypothesis is that the present price of a security encompasses all the presently available information – including past prices – concerning this security and prices tend to move only if and when a fresh information about the security percolates into the market. Even the seminal work of Fischer Black & Myron Scholes in the pricing of contingent financial claims (that constitutes the cornerstone of contemporary valuation theory) presupposes that the stock prices follow a geometric Brownian motion, the two principal attributes whereof are that:-

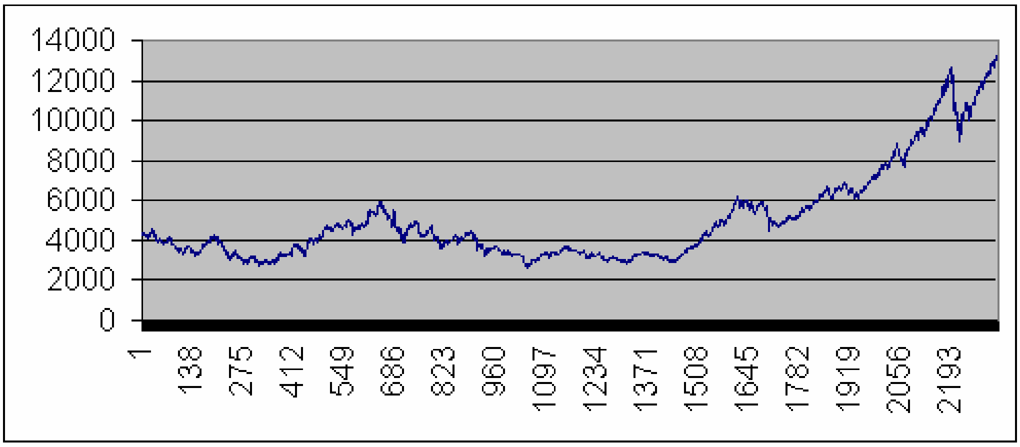
(i) the set of stock prices $S(t), 0 \le t < \infty$ constitute a geometric Brownian motion if, $\forall s,t \ge 0$, the random variable $\frac{S(s+t)}{S(t)}$ is independent of all prices up to time t;

(ii) $\ln\left[\frac{S(s+t)}{S(t)}\right]$ is a normally distributed random variable $\forall s,t \ge 0$ with mean μt and variance $t\sigma^2$ where μ and σ constitute the parameters defining the geometric Brownian motion.

(iii) It follows from (i) & (ii) that the probabilities of the ratio of the price at time in the future to the present price will not depend on the present price. Additionally, if μ and σ are known, then it is only the present price – and not the history of past prices – that affects the expectations of future prices. Specifically, we have, $E\left[S(t)\right]=S(0)e^{\left(\mu+\frac{1}{2}\sigma^2\right)t}$.

In the empirical study that constitutes the substratum of this paper, we test the hypothesis that future price movements are independent of past movements i.e. the stock market logarithmic returns follow a normal distribution in the Indian capital markets. We also examine whether memory effects of any significant duration subsist in these markets. We assume the 30 security BSE SENSEX market index as the proxy for the Indian stock market and conduct the analysis of the Sensex over the period from July 01, 1997 to November 10, 2006. consisting of 2,317 observations. A chronological plot of the Sensex values for the above period is given in Figure 1.

Figure 1



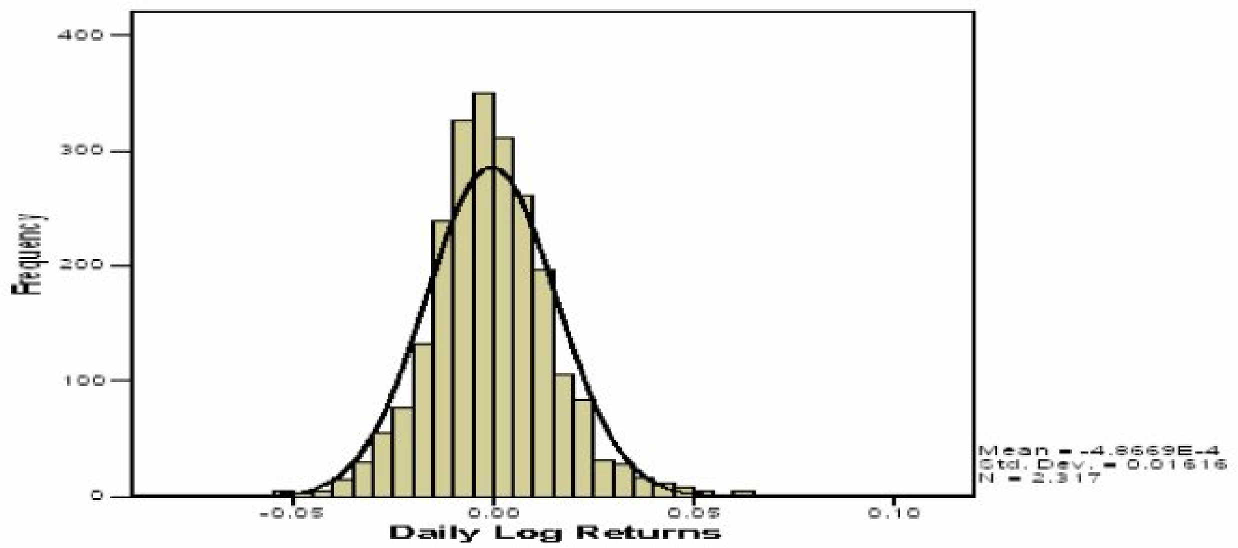
The mean and standard deviation of the daily logarithmic returns of the prices constituting the sample was found to be 0.000486691and 0.016158843 respectively. To test these returns for normality, the number and percentage of observations in the various 0.5σ intervals were calculated and compared with the corresponding values for the standard normal distribution $\mathrm{N}(0,1)$. The results are tabulated in Table 1 below:-

Table 1:

Interval	No. of Observations	% of Total	Corresponding value for $\mathrm{N}(0,1)$
$x \le \overline{x}-2\sigma$	74	0.031938	0.0228
$\overline{x}-2\sigma < x \le \overline{x}-1.5\sigma$	53	0.022874	0.044
$\overline{x}-1.5\sigma < x \le \overline{x}-\sigma$	157	0.06776	0.0919
$\overline{x}-\sigma < x \le \overline{x}-0.5\sigma$	331	0.142857	0.1498
$\overline{x}-0.5\sigma < x \le \overline{x}$	497	0.214502	0.1915
$\overline{x} < x \le \overline{x}+0.5\sigma$	553	0.238671	0.1915
$\overline{x}+0.5\sigma < x \le \overline{x}+\sigma$	385	0.166163	0.1498
$\overline{x}+\sigma < x \le \overline{x}+1.5\sigma$	139	0.059991	0.0919
$\overline{x}+1.5\sigma < x \le \overline{x}+2\sigma$	82	0.035391	0.044
$\overline{x}+2\sigma < x$	46	0.019853	0.0228

A histogram corresponding to the above data is placed at Figure 2. It is clear from the histogram that the assumption of normality of log-returns is, at best, questionable. We pursue the analysis further in the next section.

Figure 2



3. Testing for Memory Effects

As mentioned above, one of the crucial fallouts of the Efficient Market Hypothesis and/or of the assumption of geometric Brownian motion as a model of stock prices is the total absence of any kind of memory effects in the return processes. The above histogram does not provide any level of conclusive evidence for or against the existence of memory effects since it breaks up the range of data values into intervals and then plots the number of data values that fall in each interval. It does not, therefore, provide information about possible dependencies among the data.

To examine the possible existence of dependencies, the daily logarithmic returns (x) are classified into one of six possible class intervals A, B, C, D, E & F viz $x \le \overline{x} - 2\sigma$ (A), $\overline{x} - 2\sigma < x \le \overline{x} - \sigma$ (B), $\overline{x} - \sigma < x \le \overline{x}$ (C), $\overline{x} < x \le \overline{x} + \sigma$ (D), $\overline{x} + \sigma < x \le \overline{x} + 2\sigma$ (E) and $\overline{x} + 2\sigma < x$ (F). The observations falling into each of these class intervals are, then, split up on the basis the daily logarithmic return on the next following day. We, thus get a 6×6 square matrix (Table 2), the $(i, j)^{th}$ element of which would be the observation that has a logarithmic return falling in the i^{th} class on day n and j^{th} class on day $n+1$. Now, if the price evolution follows a geometric Brownian motion then tomorrow's state should not depend on today's state. In other words, the subset of observations comprising every row should behave as if they are extracted from a normal population. We proceed to test this hypothesis by the well-known χ^2 test, where the expected frequencies are the corresponding values from a normal population. Table 3 summarizes the computations (In Table 3, we have replaced absolute values of various cells of Table 2 by the corresponding percentages. The expected frequencies, also in percentages, are given in parenthesis).

Table 2

	A	B	C	D	E	F	
A	12	9	15	16	12	10	74
B	13	28	64	74	24	7	210
C	27	89	329	315	58	10	828
D	12	58	339	430	85	14	938
E	6	21	71	86	33	3	220
F	4	5	10	16	8	3	46

Table 3

	A (2.28)	B (13.59)	C (34.13)	D (34.13)	E (13.59)	F (2.28)	χ^2
A	16.21622	12.16216	20.27027	21.62162	16.21622	13.51351	151.40
B	6.190476	13.33333	30.47619	35.2381	11.42857	3.333333	7.97
C	3.26087	10.74879	39.7343	38.04348	7.004831	1.207729	6.08
D	1.279318	6.183369	36.14072	45.84222	9.061834	1.492537	10.39
E	2.727273	9.545455	32.27273	39.09091	15	1.363636	2.63
F	8.695652	10.86957	21.73913	34.78261	17.3913	6.521739	32.06

The overall value of χ^2 is found to be 210.54.

It is clear from Table 3 that in both the tails of the distribution representing extremal cases, there exist very significant memory effects over the one-day period and the distributions are well distorted from the normal distribution – the distortion is particularly massive in the left tail. However, over the intermediate range, the χ^2 values are much closer to the tabulated values, although still well above them even at a 1% significance level establishing that some degree of memory effects do exist even in this range. The histograms of each class interval are placed in Figures 3 (A) to 3 (F)

Figure 3(A)

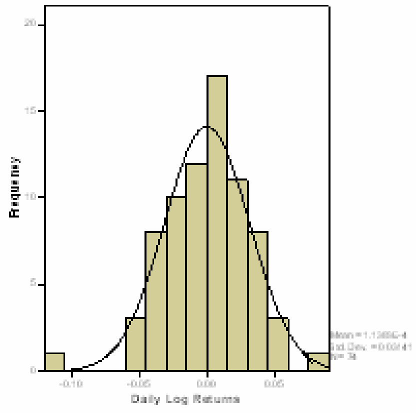


Figure 3(B)

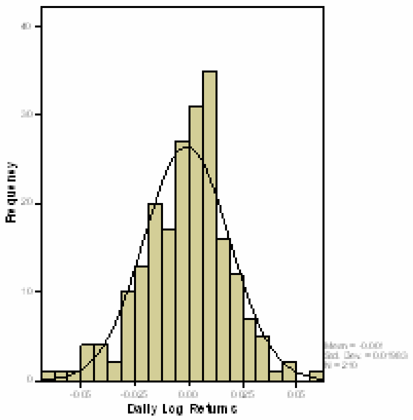


Figure 3(C)

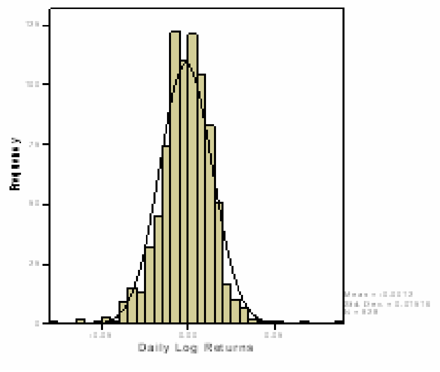


Figure 3(D)

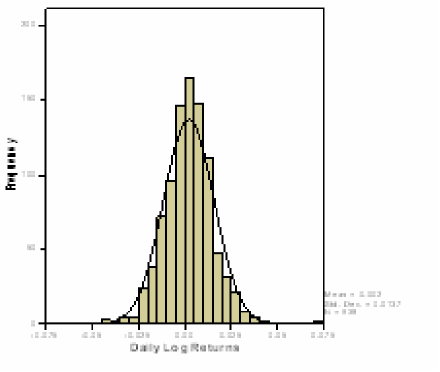


Figure 3(E)

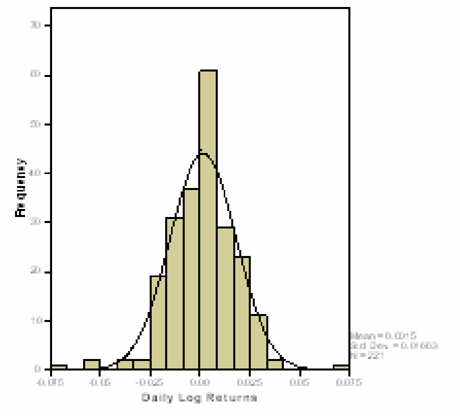
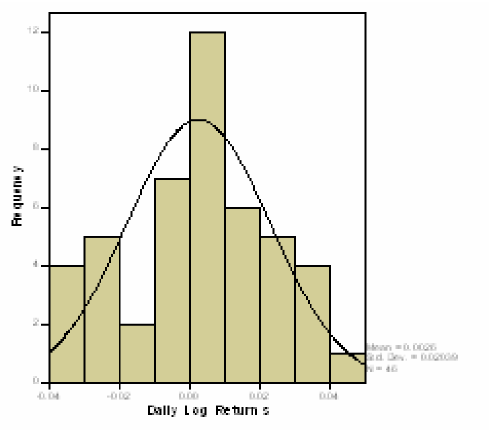


Figure 3(F)



To further corroborate the departure of real data from the geometric Brownian motion and the subsistence of significant memory effects we have also performed the one-way ANOVA test to test the hypothesis that all the six data sets describe normal random variables having the same mean and variance. The results of the analysis are tabulated below in Table 4:-

Table 4

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.005	5	.001	4.184	.001
Within Groups	.600	2311	.000		
Total	**.605**	**2316**			

The computations provide an F-statistic value of 4.184 that implies that the differences in the means of the six subsets are significant at the 0.05 significant level. In fact, the F-statistic value of 4.184 implies statistical acceptability of the hypothesis that the subsets A, B, C, D, E & F are normally distributed with the same mean only at a 0.001 level of significance.

4. Chaos [23-28] & Stock Markets

Chaos, as a physical phenomenon, has attained recognition relatively recently. Its origin, in its modern form, may be traced to the revolutionizing work of the master French mathematician Henri Poincare in the 1890s', on the mathematical aspects of planetary motion, treating it as a three-body problem. Through the use of topological methods, he established that there is no simple solution to the three-body problem. During the course of his analysis, he realized that if one takes two different readings on the position of a planet, then, irrespective of the proximity of the two readings, the orbits of the planets might separate away from each other, after enough time. Hence, accurate prediction of the orbit of any planetary body was impossible. Chaos was, thus, born.

The most apt yet striking manifestation of chaos is summarized in the following statement attributed to Edward Lorenz:-

"The flapping of butterfly wings in Rio de Janeiro could bring on a tornado in Texas several weeks later!"

This is what Edward Lorenz concluded one fine day when he was running a mathematical model of the weather on his computer. It so happened that in order to recheck some results on his weather forecasting model, he decided to re-input his data from the earlier printouts and run the program again. The results were quite inconceivable – although the immediate values of the variable were identical, major divergences surfaced as the run steps was extended – in fact, no significant resemblance was observed between the results of the two runs after a sufficiently long period. Thus, starting from nearly the same initial conditions, weather patterns were produced that grew further and further apart until all association disappeared.

On investigation, the cause of this highly paradoxical scenario was traced to a very trivial matter – while the printer had printed data upto six decimal points which constituted the data fed for the second run, the computer had calculated data upto eight decimals. The data that was fed for the second run was, therefore, minutely different from the data that was used in the first run. Amazing as it may sound, it was these minute differences that manifested themselves as gigantic divergences in the output – this, indeed, is chaos.

This property that manifests itself through sensitivity to initial conditions with a consequential unpredictability is generally termed as Chaos. The discovery of chaos has destroyed the deterministic image of the modern world leading to new directions of research and providing a fillip to the ergodic description of systems.

Chaos provides a link between deterministic systems and random processes. In a deterministic system, chaotic dynamics can amplify small differences, which in the long run produce effectively unpredictable behavior. On the other hand, it is possible that not all random-looking behavior is the product of complicated interactions and hence, may well be tractable in the deterministic framework. The existence of non-linearities in only a few degrees of freedom are sufficient to generate chaotic motion. In such cases, it is possible to model the system behavior deterministically and to make short-term predictions that are far better than those that would be obtained from a linear stochastic model. Chaos is thus a double-edged sword: it implies that even approximate long-term predictions may be impossible, but that very accurate short-term predictions may be possible. Hence, chaos has both good and bad implications for the prediction problem.

Decision making of all kinds, including investments in the capital markets, rests on our ability to predict the future. However, business and, indeed, life in general, is not predictable. Conventionally, in decision theory, this lack of predictability is explained by factors such as lack of information or the limitations of prediction techniques. Chaos theory, however, provides a radically opposite explanation, in that it accepts unpredictability as an inherent attribute of a wide range of phenomena, so that, forecasting may be an entirely futile and wasteful exercise.

Prediction and forecasting have, hitherto, relied essentially on various linear models like regression, linear programming, capital budgeting and so on. It is, however, now established beyond doubt that all fundamental processes of Nature have various degrees of non-linearity. In fact, Chaos is a manifestation of the non-linearities inherent in a system in so far as such unpredictable phenomena are forbidden in linear systems by the very virtue of their linearity.

A corollary to this ubiquitous non-linearity is the high degree of approximation incumbent in all the contemporary decision making processes. Chaos theory emphasizes that because of this sensitivity to initial conditions, many events simply cannot be predicted, because it would be impossible to know and monitor all the variations that might have a significant effect on the outcomes.

A compact, concise and universally acceptable definition of Chaos has, hitherto, eluded the scientific community. However, the following are conventionally accepted as the inherent characteristics of a chaotic system:-

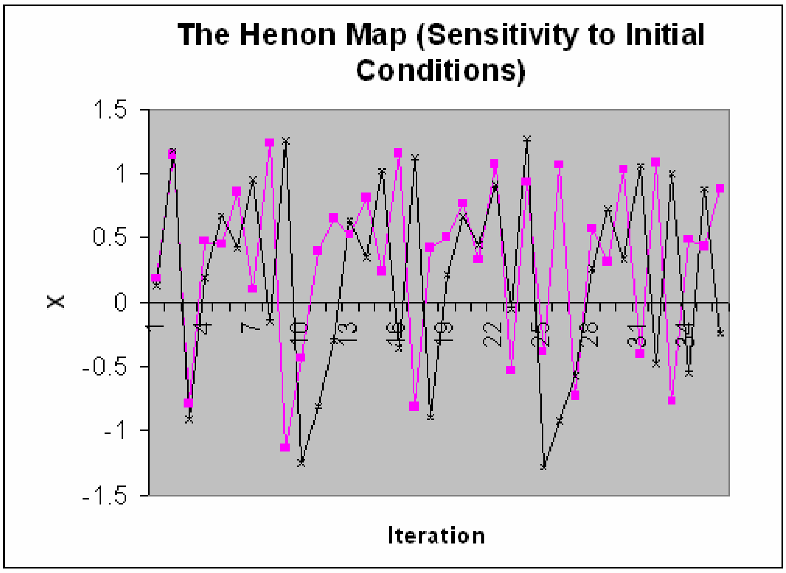
- Exponential divergence of neighboring trajectories in phase space;
- Sensitive dependence on initial conditions;
- Existence of fractal dimensions;
- Critical levels and bifurcations at which the system's behavior radically changes;
- Time dependent feedback.

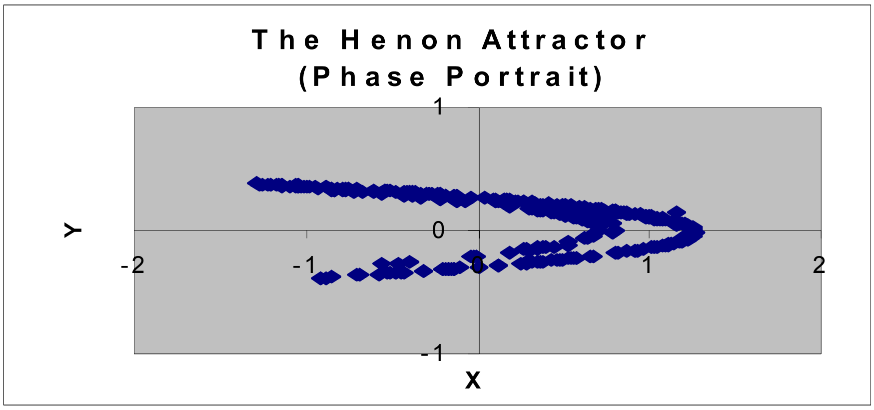
As had happened in the case of Edward Lorenz, chaotic systems are highly sensitive to initial conditions insofar as minor differences tend to get magnified manifold with the evolution of the system. This is illustrated in the Henon Map, defined by the following set of simultaneous difference equations:-

$$x_{(t+1)} = 1 + y_t - a x_t^2$$

$$y_{(t+1)} = b x_t$$

Figure 4: The Henon Map (Sensitivity to Initial Conditions)





While the origin of this sensitive dependence may be attributed to the existence of time dependent feedback mechanisms, the implications are devastating. Unpredictability becomes an inherent attribute and long term forecasting becomes a futile exercise. Marginally small errors in data collection would manifest themselves magnified manifold in forecasted output. For all we know, even with the best available measurement devices, error free measurement is impossible, a fundamental lower limit being imposed by Heisenberg's Uncertainty Principle.

Chaos theory propounds the adoption of a radically new perspective to forecasting. It emphasises the need to acknowledge the true dimensions of uncertainty in its absoluteness and to discard the conventional and traditional so called "rationalistic" models like the Efficient Market Hypothesis. Chaos theory recognises the existence of disorder, discontinuities and randomness as inherent properties or norms rather than as aberrations. Consequent to the acceptance of unpredictability as an inherent property, chaos theory tends to dwell heavily on the necessity of development of adequate "fire fighting" mechanisms as an indispensable part of planning and forecasting.

Several studies [1,4,8-11] adopting largely diverse and independent approaches have established the existence of the following characteristics in the behavior of stock markets:-

- Long term correlation and memory effects
- Occasional existence of erratic markets
- Existence of fractal dimensions in stock market time series of returns
- Less reliable forecasts with increase in the horizon thereby establishing the probable existence of chaotic behavior of stock markets.

As mentioned above, the phenomenon of chaos is a manifestation of the non-linearities intrinsic to the dynamic equations of motion that govern the time development of a system. In the next section, we look at some approaches that have been devised to examine the existence or otherwise of nonlinear and chaotic behavior in time series data that represents the evolution of discrete phenomena like the prices of stocks in capital markets.

5. Rescaled Range Analysis & Computation of Hurst's Exponent

Variance has been, traditionally, used in one guise or another as the statistical measure of risk. Variance measures the probability that an observation will be a certain distance from the average observation. The larger this number, the wider the dispersion. Wide dispersion would mean that there is a high probability of large swings in returns. The security is risky. However, the use of variance as a measure of risk inherently assumes that the underlying system is random. If the observations are correlated, then the usefulness of variance as a measure of risk is considerably weakened. We illustrate our point by an example. We consider two possible series of stock market returns, say A & B:-

Table 5

Observation	**A**	**B**
1	0.02	0.01
2	-0.01	0.02
3	-0.02	0.03
4	0.02	0.04
5	-0.01	0.05
6	0.02	0.06
Standard Deviation	0.17	0.0171

A is a trendless series while B has a clear trend. Both have almost the same standard deviation. The two stocks with virtually identical risk (as measured by the standard deviation) have vastly differing return characteristics. The obvious fallacy is that both series are not normally distributed, but then the same is the case with the stock markets. As mentioned above, numerous studies have shown the non-random character of the stock market returns, thereby questioning the usefulness of variance as a comparative measure of risk.

A time series will be truly random when it is influenced by a number of events that are equally likely to occur i.e.. e. the system has a large number of degrees of freedom. In a non-random series the data will clump together to reflect the correlation inherent in its influences and the time series will be a fractal.

The stock markets are modelled as a process that happens in time. As is the case with most systems modelling, this process is treated either as a discrete static process or a continuous random process. However, neither assumption entirely gels with reality and nor extreme is a complete and sophisticated treatment of the subject. The commonality underlying both these assumptions is that they are linear i.e. either they are always static or always random. Time either does not effect the system or does so at a uniform rate.

Most financial returns, including stock returns have shown deviation from Gaussian behaviour at short time scales with the variance not scaling with the sq. root of timescale, an attribute that is symptomatic of the possible existence of power law distributions. A useful measure of quantifying deviations from the Gaussian distribution is the Hurst's exponent. If a population is Gaussian, a Hurst's exponent of 0.5 is mandated. Empirical evidence, however, shows that the Hurst's exponent for typical stock market data is around 0.6 for small timescales of about a day or less and tends to approach 0.5 asymptotically with the lengthening of the timescales. Empirical evidence also demonstrates the existence of memory effects, particularly in stock price volatilities that show long-term memory effects with lag-s autocorrelations. Further, these effects tend to fall off according to a power law rather than exponentially.

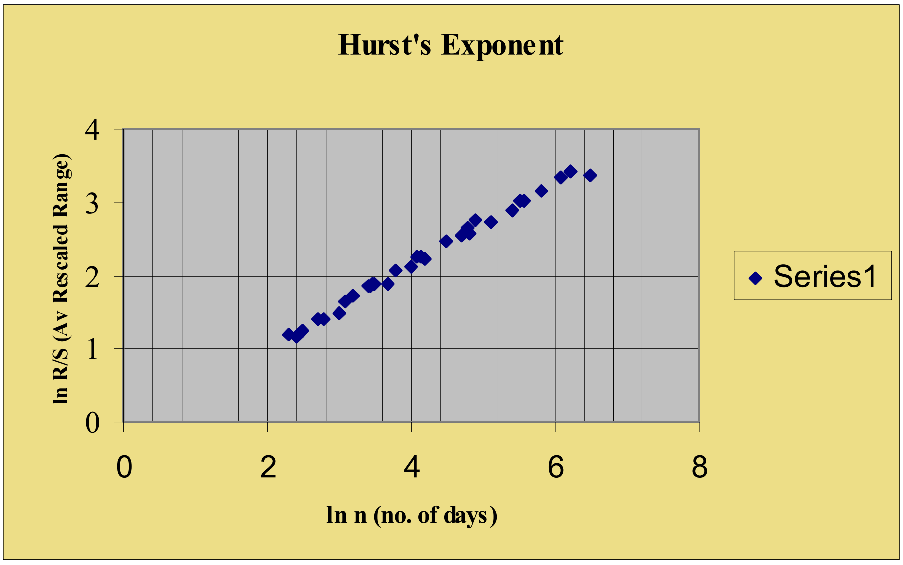
In our study, we have performed the Rescaled Range analysis of the logarithmic returns on the BSE Sensex for the period from July 01, 1997 to November 10, 2006 consisting of 2,317 observations in the manner provided in Ref [21-22]. The results are tabulated in Table 6 below:-

Table 6

Days	Average Rescaled Range	Days	Average Rescaled Range	Days	Average Rescaled Range
10	3.273144	32	6.637159	120	14.08
11	3.230032	33	6.635194	124	13.13
12	3.500606	40	6.585	132	15.54
15	4.048635	44	7.99	165	15.21
16	4.035323	55	8.23	220	17.75
20	4.455719	60	9.61	248	20.76
22	5.146307	62	9.48	264	20.7
24	5.631192	66	9.37	330	23.4
30	6.350008	88	11.6	440	28.09
31	6.306971	110	12.68	496	30.77

The relationship between the Rescaled Range R/S and the Hurst's Exponent H is given by $R/S = A\times(n)^H$ or equivalently $H = \frac{\ln(R/S) - \ln A}{\ln n}$. Hence, in order to obtain the Hurst's exponent, the logarithm of the Rescaled Range is plotted against the logarithm of the number of days, the slope of this plot (Figure 5) being the value of H. The slope and hence, the value of the Hurst's Exponent is found to be **0.58** which is commensurate with similar findings in stock markets of several other countries [21-22].

Figure 5



There is a simple mathematical link between the Fractal Dimension or the associated Hurst's Exponent H and the physical dimension. H = 1 corresponds to a perfectly persistent series representable by a unidimensional line whereas H=0.50 corresponds to random or Brownian motion, it is equal to a dimension of 1.50, a fractal or noninteger dimension halfway between a line and a plane. And where H=0, a perfectly antipersistent time series, the corresponding physical dimension is a plane or 2.

The fact that the value of the Hurst's exponent for the time series that is the subject matter of this study has been found to be 0.58 corroborates our earlier findings that stock market returns in the Indian capital markets are not random and hence, do not constitute a population that is normally distributed. Furthermore, geometric Brownian motion cannot accurately model the stock prices. There also exist significant memory effects that result in lumping of observations into some sort of clusters particularly for returns that are located in the tails of the distribution.

Not only do the quantitative tests contradict the validity of the EMH, but the assumption of "Investor Rationality" as envisaged in the EMH may also be questioned on the following grounds:-

- The EMH presupposes that all investors are risk averse. However, investors may not be risk averse in all situations. They may become risk takers in certain situations e.g. when confronted with a situation that involves perceived sure losses. For example, if asked for a trade off between a certain loss of $. 85,000 vs. a loss of $. 1,00,000 with a probability of 0.85 and a zero loss with probability of 0.15 would generally find the investor opting for the latter;
- Investors are usually more confident of their forecasts than is warranted by the available information. They have a tendency to ignore new information if it does not fit in with their current forecasts of the future;
- Investors would not normally react to trends until fully established, a phenomenon that takes some time. They will not begin to accept and extrapolate a set of circumstances until it is firmly established. They then take a decision on the basis of all the information that has accumulated thus far. In other words, reaction to information does not occur in a continuing fashion as and when it is received, but rather in discrete blocks & clumps in a cumulative fashion.
- Only when the level of information reaches a critical level, investors react to all the information received till then. Hence, memory effects subsist

- As a corollary to the above, markets lose their efficiency since all information is not reflected in prices and much of the information is ignored and accumulated till it reaches a threshold level and reaction comes later;

Acceptance of complete randomness in stock prices is beset with questions of consistency as well. One must need appreciate that the vantage point of each investor is different and also that it keeps on changing with the passage of time. That is, they have different points of view. If that is not the case, how does one explain as to why people can be rational investors and still make very different investment decisions. The fact remains that everyone's perspective is different and is varying. Rationality of investors may be construed in that they are internally self-consistent with the information that they possess. However, their decisions may seem illogical from a different informational point of view.

References

[1] Sharpe W.F., Portfolio Theory & Capital Markets, McGraw Hill, 1970.
[2] Elton E.J. & Gruber M.J., Modern Portfolio Theory,& Investment Analysis, Wiley, 1981.
[3] Ross S.M., Stochastic Processes, John Wiley, 1999.
[4] Cootner P., Ed., The Random Character of Stock Market Prices, Cambridge MIT Press, 1964.
[5] Black F. & Scholes M., The Pricing of Options & Corporate Liabilities, Journal of Political Economy, May/June, 1973.
[6] Jensen M.C., Studies in the Theory of Capital Markets, Praeger, 1972.
[7] Cox J.C. & Rubinstein M., Option Markets, Englewood Cliffs, Prentice Hall, 1985.
[8] Fama E.F., The Behaviour of Stock Market Prices, Journal of Business, 38, 1965.
[9] Fama E.F., Efficient Capital Markets – A Review of Theory & Empirical Work, Journal of Finance, 25, 1970.
[10] Fama E.F. & French K.R., The Cross Section of Expected Stock Returns, Journal of Finance, 47, 1992.
[11] Fama E.F. & Miller M.H., The Theory of Finance, Holt Rinehart & Winston, 1972.
[12] P. Gopikrishnan et al, Phys. Rev. E 60, 5305, (1999);
[13] P. Gopikrishnan et al, Phys. Rev. E 62, R4493, (2000);
[14] P. Gopikrishnan et al, Physica A, 299, 137, (2001);
[15] P. Gopikrishnan et al, Phys. Rev. E 60, 5305, (1999);
[16] V. Plerou et al, Phys. Rev. E 60, 6519, (1999);
[17] Wolf A., Swift J.B., Swinney H.L. & Vastano J.A., Determining Lyapunov Exponents From a Time Series, Physica 16D, 1985, 285.
[18] Mandlebrot B.B., The Fractal Geometry of Nature, Freeman Press, 1977.
[19] DeBoek G, Ed., Trading on the Edge, Wiley, 1994.
[20] DeGooijer J.G., Testing Nonlinearities in World Stock Market Prices, Economics Letters, 31, 1989.
[21] Peters E., A Chaotic Attractor for the S&P 500, Financial Analysts Journal, March/April, 1991.
[22] Peters E., Fractal Structure in the Capital Markets, Financial Analysts Journal, July/August, 1989.
[23] Lichtenberg, A. and Lieberman, M., Regular & Stochastic Motion, Springer, 1983.
[24] Reichl, L.E., the Transition to Chaos, Springer, 1992.
[25] Arnol'd, V.I. and Avez, A., Ergodic Problems of Classical Mechanics, Benjamin, 1968.
[26] Kornfeld, I., Fomin, S. and Sinai, Ya, Ergodic Theory, Springer, 1982.
[27] Guckenheimer, J. and Holmes, P., Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer Verlag, 1983.
[28] Zaslavsky G.M., Chaos in Dynamic Systems, Harwood Academic, 1985.