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Forecasting Volatility: Evidence from the Macedonian Stock Exchange

Zlatko J. Kovačić
School of Information and Social Sciences, The Open Polytechnic of New Zealand
Wellington, New Zealand
E-mail: Zlatko.Kovacic@openpolytechnic.ac.nz
Tel: +64-4-9135777; Fax: +64-4-913-5727

Abstract

This paper investigates the behavior of stock returns in an emerging stock market namely, the Macedonian Stock Exchange, focusing on the relationship between returns and conditional volatility. The conditional mean follows a GARCH-M model, while for the conditional variance one symmetric (GARCH) and four asymmetric GARCH types of models (EGARCH, GJR, TARCH and PGARCH) were tested. We examine how accurately these GARCH models forecast volatility under various error distributions. Three distributions were assumed, i.e. Gaussian, Student $-t$ and Generalized Error Distribution. The empirical results show the following: (i) the Macedonian stock returns time series display stylized facts such as volatility clustering, high kurtosis, and low starting and slow-decaying autocorrelation function of squared returns; (ii) the asymmetric models show a little evidence on the existence of leverage effect; (iii) the estimated mean equation provide only a weak evidence on the existence of risk premium; (iv) the results are quite robust across different error distributions; and (v) GARCH models with non-Gaussian error distributions are superior to their counterparts estimated under normality in terms of their in-sample and out-of-sample forecasting accuracy.

Keywords: Stock market, forecasting volatility, South-Eastern Europe, GARCH models, non-Gaussian error distribution, Macedonia
JEL Classification Codes: G12, C22, C52.

1. Introduction

Financial market volatility is a central issue to the theory and practice of asset pricing, asset allocation, and risk management. Though earlier financial models assumed volatilities to be constant, it is widely recognized among both practitioners and academics that volatility varies over time. This recognition initiated an extensive research program into the distributional and dynamic properties of stock market volatility. Stock volatility is simply defined as a conditional variance, or standard deviation of stock returns that is not directly observable. Since the optimal decision of investors relies on variance of returns that can change over time, it is important to model and forecast conditional variance. There are three ways to calculate volatility: using high-frequency data, implied volatility of options data and by econometric modeling. This paper focuses on the econometric modeling of volatility and family of GARCH models in particular. An excellent review of volatility forecasting can be found in Poon & Granger (2003). They reviewed the methodologies and empirical findings in more than 90 published and working papers that study forecasting performance of various volatility models. Xiao & Aydemir (2007) also provided a good overview of volatility forecasting models, highlighting the similarities and differences between them.

Emerging capital markets of the countries of former Yugoslavia are becoming increasingly important for both institutional and individual investors. However, they still remain small, fragmented and underdeveloped as Müller-Jentsch (2007) described them. For example, the market capitalization of all Western Balkan countries together amounts to just over € 50 billion (equity only) in 2006, which is equivalent to about a third of the already small Vienna Stock Exchange. What is even worse is that this small amount of market capitalization is fragmented between too many exchanges. Some countries, such as Montenegro and Bosnia and Herzegovina have even two stock exchanges.

Claessens, Djankov, & Klingebiel (2000) identify weak laws and regularities, slow progress on private sector development, a limited supply of institutional investors, and macroeconomic uncertainty as the main obstacles to stock market development in the eastern European countries. Rich source of information about the economic and political development and a basic data for each Eastern Europe and Central Asia stock exchanges is the latest report of the Federation of Euro-Asian Stock Exchanges FEAS (2007).

One of the newcomers into the family of Eastern European stock exchanges, the Macedonian Stock Exchange (hereafter MSE), was founded on September 13, 1995 and commenced trading on March 28, 1996. The MSE was founded as a non-profit joint stock company with a founding capital of € 500,000. According to the Securities Law banks and other financial institutions are eligible founders. Currently MSE has 17 members - 11 brokerage houses and 6 banks. After the mass privatization it became mandatory for a company to be listed on the MSE.

Table 1: Summary of key indicators for the Macedonian stock exchange in 2006

Indicator	
Number of listed companies	101
Market capitalization (millions US$)	1,103.94
Market capitalization/GDP ratio	17.73%
Volume (millions US$)	397.17
Turnover ratio (%)	35.98%
Index	MBI-10
Mean (in percent)	0.190
Maximum (in percent)	4.678
Minimum (in percent)	-4.325
Standard deviation (in percent)	1.083
Sharpe ratio	0.176

Source: Federation of Euro-Asian stock exchanges website (www.feas.org), annual report of the MSE and our calculation.
Note: Turnover ratio is volume divided by market capitalization. Sharpe ratio is mean return divided by standard deviation.

Macedonia has the smallest market capitalization among countries of former Yugoslavia. This is probably the main reason why Macedonia holds the last place among countries of former Yugoslavia when comparing its financial indicators from Table 1 with comparable indicators for other stock markets in the region. Stock market capitalization/GDP ratio measures the *developedness of stock market*. For Macedonia this ratio is equal to 17.73%, the lowest in the region in 2006. Next to Macedonia is Slovenia with 38.12% while Montenegro has the market capitalization/GDP ratio well above 100%.

Turnover ratio could be used to measure the efficiency of the market, but it is not a direct measure of efficiency. It measures the value of stock transactions relative to the size of the market, and is frequently used as a measure of *market liquidity*. According to this indicator Macedonia stock market is the most liquid in the region with turnover ratio equal to 35.98%. Among stock markets in countries of former Yugoslavia this ratio ranges from 4.21% (Banja Luka stock exchange) to 12.15% (Belgrade stock exchange) in 2006. Developed economies such as the United States and France, have a turnover ratio of approximately 50%, while less developed transition economies have a turnover ratio about 5%.

The idea of the Sharpe ratio is to see how much *additional return* investor is receiving for the additional volatility of holding the risky asset over a risk-free asset. The higher value of the Sharpe ratio is the better from investor perspective. Sharpe ratio in 2006 for Macedonia was about 0.176, the lowest in the region. Other stock exchanges in the region achieved value of the Sharpe ratio over 0.2 with Croatia, i.e. Zagreb stock exchange being on the top of the list with the Sharpe ratio equal to 0.236.

Since Macedonia is going to join the European Union, understanding of its stock market could be of interest to international investors. Identifying and comparing stochastic behavior of Macedonian stock market series with behavior of stock markets series of the European Union members could bring valuable information to investors helping them to optimize their portfolios and reduce the risk involved.

The purpose of this paper is to contribute to the debate by examining issues concerning the relationship between returns and volatility that have attracted considerable attention in other emerging markets of the Central and Eastern Europe. These issues have not been examined so far for the MSE, and the paper attempts to fill the gap by addressing the following questions:

- What are the stylized facts characterizing the behavior of MSE stock returns?
- What has been the impact of conditional volatility on stock returns, and is there evidence of significant risk premium and leverage effects?
- How robust is the relationship between returns and conditional volatility to the change of the model specification and assumed error distribution?
- Which conditional volatility model outperform other models in term of in-sample and out-of-sample forecasting accuracy?

The remainder of the paper is structured as follows. Section 2 provides a brief literature review, focusing on stylized facts and volatility of emerging stock markets in the Central and Eastern European countries. The alternative GARCH models are briefly examined in Section 3. Section 4 provides data description. Empirical results are presented in Section 5, while Section 6 concludes with a summary of the main findings and implications.

2. Literature Review

2.1. Stylized Facts of the Financial Time Series

Since the early work of Mandelbrot (1963) and Fama (1965), researchers have documented empirical regularities regarding prices, returns, and volatilities of financial time series. Due to a large body of empirical evidence, many of the regularities can be considered stylized facts. The most common stylized facts are the following:

1. **Volatility tends to cluster.** Volatility exhibits persistence that is, large return innovations of either sign tend to be followed by large innovations, or periods of high volatility with periods of high volatility and periods of low volatility are followed by periods of low volatility. This implies that volatility could be used as a predictor of volatility in the next periods. As an indication of volatility clustering, squared returns often have significant autocorrelations.
2. **Volatility is mean reverting.** This characteristic means that there is a normal level of volatility and eventually volatility will return to that level.
3. **Return distributions have heavy tails with narrower and higher peak.** Having heavy tails means that extreme returns occur more frequently than implied by a normal distribution. Distributions with such characteristics are called leptokurtotic distributions.
4. **Asymmetric reaction on "good" and "bad news".** Volatility tends to react differently on arrival of "good" and "bad news", i.e. positive and negative innovations. Black (1976) notes

the tendency for negative innovations to generate greater volatility in future periods compared with positive innovations of the same magnitude, a phenomenon that he refers to as the "leverage effect".

A good volatility model should be able to capture and reproduce most, if not all of these stylized facts. Stylized facts of the financial time series were analyzed by, amongst others, Cont (2001, 2005, 2007), Guillaume et al (1997), Kirchler & Huber (2005), Krivoruchenko, Alessio, Frappietro & Streckert (2004), Malmsten & Teräsvirta (2004) and Rydberg (2000).

2.2. Research About Volatility in the Countries of Former Yugoslavia

While the stock markets volatility in developed countries has been thoroughly investigated there is less empirical research on the stock markets volatility in transition economies of Eastern Europe. The main reason was a complete lack of data or too short stock market time series for any thoughtful analysis. The stock markets in Eastern European countries were established mainly in early nineties. The Western Balkan stock markets were established even later with reliable data for the last 4-5 years only. The following list gives the main research topics covered as well as the selection of empirical studies analyzing mostly Central and East Europe stock markets:

1. Modeling and forecasting volatility in Central and Eastern European countries (Anatolyev, 2006; Anatolyev & Shakin, 2006; Égert & Koubaa 2004; Grambovas, 2003; Hasan & Quayes, 2005; Kasch-Haroutounian & Price, 2001; Murinde & Poshakwale, 2001; Patev & Kanaryan, 2006; Poshakwale & Murinde, 2001; Shields, 1997a, 1997b; Shin, 2005; Sian, 1996)
2. Seasonal anomalies or calendar effects on European stock market volatility (Ajayi, Mehdian & Perry, 2004; Apolinario, Santana, Sales & Caro, 2006; Chukwuogor-Ndu, 2006; Tonchev & Kim, 2004)
3. Volatility transmission or spillovers between European stock markets (Baele, Crombez & Schoors, 2003; Dumitru, Mureşan & Mureşan, 2005; Égert & Kočenda, 2005; Gelos & Sahay, 2000; Inzinger & Haiss, 2006; Jochum, Kirchgässner & Platek, 1999; Kanas, 1998; Morana & Beltratti, 2002; Onay, 2006; Patev & Kanaryan, 2006; Patev, Kanaryan & Lyroudi, 2006; Scheicher, 2001; Syllignakis & Kouretas, 2006)
4. Efficiency of Eastern European stock markets (Harrison & Paton, 2005; Rockinger & Urga, 2000; Todea & Zoicaş-Ienciu, 2005)
5. Interaction between real sector and stock market (Cihak & Janaček, 1997).

Empirical studies on the stock markets in Central and East Europe listed above were mostly based on some variation or extension to the basic ARCH (Engle, 1982) and GARCH models (Bollerslev, 1986).

We reviewed 19 empirical studies on research about volatility in the countries of former Yugoslavia from various journals and working paper series. In general, we focused on papers analyzing not just volatility forecasting, but also other issues related to volatility of stock market indices. These papers are not necessarily using the same methodological framework adopted in this study. The reason for considering wider list of empirical papers is that with a few exceptions, research on volatility forecasting in the financial markets of the countries of former Yugoslavia does not exist.

As far as this author knows, among countries of former Yugoslavia only Slovenian and Croatian stock exchanges were subject to rigor analysis using the same or similar methodological approach adopted in this paper. They were the first stock exchanges set up among countries of former Yugoslavia. Thus far there has been no empirical study of the stochastic behavior of Bosnia and Herzegovina and Montenegro stock markets and only a few for Serbia and Macedonia. In the following we will briefly discuss and summarize the studies under review. A comprehensive overview of the research about volatility in the countries of former Yugoslavia is given in the Appendix.

One of the first analyses of the Croatian stock market was undertaken by Šestović & Latković (1998). They used the main Croatian stock market index and a few company's indices to estimate GARCH(1,1) model and illustrate how this model can be used in volatility forecasting. Similar objectives and results were presented in Latković (2001, 2002) and Levaj, Kamenarić, Mišković & Mokrovčak (2005). For a Croatian exchange rate series Posedal (2006) found that the nonlinear GARCH models better describes short-run dynamics, while Anatolyev (2006) rejected conditional mean independence in the volatility model for Croatian stock market. Žiković (2006a, 2006b) successfully applied VaR methodology and historical simulation on the Croatian stock market indices in an effort to measure Value-at-Risk.

Calendar effects and their impact on the conditional volatility were also subject of investigation for Croatian stock market. Ajayi, Mehdian & Perry (2004) did not found day-of-the-week effect, while Fruk (2004) rejected hypothesis of seasonal unit root in Croatian index. When investigating volatility transmission or spillovers between Croatian stock markets and other markets in the region and Europe the mixed results were obtained. Onay (2006) used a cointegration test, but did not found a long-run relationship between Croatia and other economies. However, the causality test found a causal flow from European indices to Croatian index. This is an opposite result to the result presented in Samitas, Kenourgios & Paltalidis (2006) who discovered equilibrium relationships, i.e. linkages between developed and stock markets in transitional economics (Croatia, Serbia and Macedonia) by using Markov switching regime regressions. There was only one more study which was using Belgrade stock exchange data to check whether the stylized facts exist. Miljković & Radović (2006) discovered the main commonly known stylized facts in the Serbian stock market data.

Mean predictability in the volatility model for Slovenia was not detected in Slovenian index (Anatolyev, 2006), while Égert & Koubaa (2004) found that sum of parameters in a simple GARCH(1,1) for Slovenia is over 1. However, nonlinear GARCH models such as GJR and QGARCH reasonably well modeled Slovenian stock market index. Žiković (2007) shown that use of common VaR models to forecast VaR is not suitable for transition economies such as Slovenia.

Hasan & Quayes (2005) tried to identify the level of integration between Slovenian and European financial markets. Similarly to Croatia they discovered no long-run relationships between Slovenia and nine other countries considered. However, the impact of other stock markets or external events can't be completely ruled out. Syllignakis & Kouretas (2006) identified what was the impact that the Russian crisis had on the stock markets in other countries (including Slovenia) by using multivariate version of the GARCH model, i.e. dynamic conditional correlation GARCH. They discovered that conditional volatility increased in case of Slovenia over two times during the Russian crisis.

Calendar effects on volatility of Slovenian stock market were found. Ajayi, Mehdian & Perry (2004) identified day-of-the-week effect in Slovenian index (negative Tuesday and positive Thursday and Friday effects). The same effects were investigated by Tonchev & Kim (2004) who found weak evidence for the day-of-the-week effect in mean in opposite direction, i.e. reverse effects in positive returns. By using GARCH model they identified calendar effects in the conditional variance such as January effect, monthly seasonality in variance and the reverse half-month effect. Finally, Deželan (2000) rejected a weak form of efficiency hypothesis for the Slovenian stock market.

3. GARCH-Type Models

3.1. Symmetric GARCH-in-Mean Model

The starting model used in this paper is based on an extension of the basic GARCH model proposed by Engle, Lilien, & Robins (1987) so that the conditional volatility can generate a risk premium which is part of the expected returns. An AR(2)-GARCH(1,1)-M model is specified with the following two equations:

Mean equation: $$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \lambda\sigma_t + \varepsilon_t, \quad (1)$$

Variance equation: $$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \quad (2)$$

where r_t is the stock market return, and ε_t is a Gaussian innovation with zero mean and a time-varying conditional variance σ_t^2. Three coefficients λ, α and β are non-negative constants. The coefficient α in the variance equation measures the reaction of volatility on market movements. Higher values for this coefficient would generate more "spiky" diagram of returns, i.e. conditional volatility would show large reaction and low persistence. The coefficient β in the variance equation measures the persistence of volatility. Higher values for this coefficient means that innovations to conditional variance will take longer to die out, i.e. conditional volatility would show low reaction and large persistence. Ling and McAleer (2002a) established the necessary and sufficient condition for the existence of the second moment of ε_t for GARCH(1,1) model: $\alpha+\beta<1$, the unconditional variance is $\omega/(1-\alpha-\beta)$ and kurtosis is greater than 3 (i.e. leptokurtic distribution).

The coefficient λ in the mean equation measures the risk premium describing the nature of the relationship between stock market returns and volatility. If this coefficient is positive we would expect that investors are compensated with higher returns for taking the higher risk (volatility). If the coefficient λ is negative that would mean investors are getting less than expected despite taking higher risk.

The standard GARCH model is symmetric in its response to past innovations. Since good news and bad news may have different effects on the volatility we considered several alternative GARCH models in an attempt to capture the asymmetric nature of volatility responses. ARCH-type models, their specification, estimating and testing have been reviewed by, amongst others, Bera & Higgins (1993), Bollerslev, Chou & Kroner (1992), Bollerslev, Engle & Nelson (1994) and Palm (1996).

3.2. Asymmetric GARCH Models

It was observed that volatility tend to increase more when the stock market index was decreasing than when the stock market index was increasing by the same amount. As discussed by Cappiello, Engle & Sheppard (2003), asymmetric volatility can be explained by two models: leverage effect and time-varying risk premium (volatility feedback). According to Black (1976) reason for such phenomenon might be that when the equity price falls the debt remains constant in the short term, so the debt/equity ratio increases. The firm became more highly leveraged and future of the firm becomes more uncertain. The equity price therefore becomes more volatile. An alternative explanation of the asymmetric volatility responses is based on the time-varying risk premium (Campbell and Hentschel, 1992; Wu, 2001). According to them, if volatility is priced, an expected increase in volatility raises the required return on equity, leading to an immediate stock price decline. Bekaert and Wu (2000) shown, when combining these two explanations in an empirical model, often the coefficient linking volatility to expected return is insignificant, and the sign is different depending on the study. Also, that the leverage effect alone does not adequately explain the changes in volatility after a decrease in the asset price. Finally, a third explanation, described as following-the-herd effect (De Goeij & Marquering, 2004) is based on a psychological behavior. Investors might pay less attention to the market fundamentals during a stock market crash, and therefore sell their stocks if everybody else is selling. The negative relationship between stock returns and volatility was further discussed in Jinho, Chang-Jin & Nelson (2007).

Since the symmetric GARCH model is unable to account for the leverage effects observed in stock returns, asymmetric GARCH models were proposed that enable conditional variance to respond asymmetrically to rises and falls in innovations.

3.2.1. Exponential GARCH Model

An asymmetric model allows the possibility that unexpected drop in price (arrival of the "bad news") has a larger impact on future volatility than an unexpected increase in price (arrival of the "good news") of similar magnitude. Nelson (1991) proposed an exponential GARCH or EGARCH(1,1) model given by

$$\log\sigma_t^2 = \omega + \alpha\left[\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)\right] + \beta\log\sigma_{t-1}^2 + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (3)$$

where α, β and γ are constant parameters. It is expected that $\gamma < 0$, "good news" generate less volatility than "bad news", where γ reflects the leverage effect. When ε_{t-1} is positive, i.e. there is a "good news", the total contribution to the volatility of innovation is $\alpha(1+\gamma)|\varepsilon_{t-1}|$. In opposite case, when ε_{t-1} is negative, i.e. there is a "bad news", the total contribution to the volatility of innovation is $\alpha(1-\gamma)|\varepsilon_{t-1}|$. The EGARCH model specifies conditional variance in logarithmic form, which means that there is no need to impose estimation constraints in order to avoid negative variance. The EGARCH model is asymmetric as long as $\gamma \neq 0$. Though Nelson (1991) originally assumed that the ε_t follows a Generalized Error Distribution (GED), we have estimated this model using three different distributions: normal, Student$-t$ and GED.

3.2.2. GJR Model

This model is also known as GJR model proposed by Glosten, Jagannathan & Runkle (1993). Variance equation in a GJR(1,1) model is given by

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 \quad (4)$$

where α, β and γ are constant parameters and I_t is an indicator dummy variable that takes the value 1 if $\varepsilon_{t-1} < 0$ and zero otherwise. The impact of ε_t^2 on the conditional variance σ_t^2 in this model is different when ε_t is positive or negative. The negative innovations ("bad news") have a higher impact than positive ones. When ε_{t-1} is positive, the total contribution to the volatility of innovation is $\alpha\varepsilon_{t-1}^2$; when ε_{t-1} is negative, the total contribution to the volatility of innovation is $(\alpha+\gamma)\varepsilon_{t-1}^2$. We would expect γ to be positive, so that the "bad news" has larger impacts. In that case we say there is a leverage effect. The GJR(1,1) model is asymmetric as long as $\gamma \neq 0$. Ling & McAleer (2002b) established the regularity condition for the existence of the second moment of GJR(1,1) model, which is $\alpha + \beta + \gamma / 2 < 1$.

3.2.3. Threshold GARCH model

Another asymmetric variant of GARCH model is the threshold GARCH (TGARCH) model proposed by Zakoïan (1994). It is similar to the GJR, but models the conditional standard deviation instead of the conditional variance:

$$\sigma_t = \omega + \alpha\varepsilon_{t-1} + \beta\sigma_{t-1} + \gamma I_{t-1}\varepsilon_{t-1} \quad (5)$$

where α, β and γ are constant parameters and I_t is an indicator dummy variable that takes the value 1 if $e_{t-1} < 0$ and zero otherwise. Similarly to GJR model when ε_{t-1} is positive, the total contribution to the volatility of innovation is $\alpha\varepsilon_{t-1}$; when ε_{t-1} is negative, the total contribution to the volatility of innovation is $(\alpha+\gamma)\varepsilon_{t-1}$. We would expect γ to be positive, so that the "bad news" has larger impacts. In that case we say there is a leverage effect. The TGARCH model is asymmetric as long as $\gamma \neq 0$.

3.2.4. Power GARCH model

Ding, Granger & Engle (1993) proposed a class of models which encompasses a few other GARCH models. This class of models is called Power GARCH (PGARCH) models. Variance equation in PGARCH(1,1) is given by

$$\sigma_t^\delta = \omega + \alpha(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta \quad (6)$$

where α , β and γ are constant parameters and $\delta > 0$, and $|\gamma| < 1$. Parameter δ could be fixed in the PGARCH models before estimation. Usually choices for this parameter are $\delta = 1$ (then the PGARCH model is robust to outliers) and $\delta = 2$. Coefficient δ . plays the role of a Box-Cox power transformation of the conditional standard deviation process. The PGARCH model embeds GARCH, GJR and a few other ARCH-type models. For example, when $\delta = 2$, and $\gamma = 0$, PGARCH reduces to a GARCH model. When $\delta = 2$ PGARCH reduces to a GJR model. When $\delta = 1$ PGARCH reduces to a TGARCH model.

3.3. Alternative conditional distributions and estimation

To completely specify a GARCH-type model an assumption about the error distribution ε_t should be made. As it was mentioned before, it is more appropriate to assume that the errors have a heavy tailed distribution rather than Gaussian distribution. Beside the Gaussian conditional distribution of the error term ε_t two alternative non-Gaussian distributions are considered: Student $-t$ distribution and generalised error distribution (GED).

Standardized Student $-t$ distribution for $z_t = \varepsilon_t / \sigma_t$, standardized errors can be expressed as

$$f(z_t \mid \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\left(1+\frac{z_t^2}{\nu-2}\right)^{\frac{\nu+1}{2}}} \tag{7}$$

where $\Gamma(\cdot)$ is the gamma function, and $\nu > 2$ is the shape parameter.

Generalized error distribution was suggested to be used in GARCH models by Nelson (1991). It can be expressed as

$$f(z_t \mid \nu) = \frac{\nu}{\lambda_\nu \cdot 2^{(\nu+1)/\nu}\Gamma(1/\nu)} \exp\left(-\frac{1}{2}\left|\frac{z_t}{\lambda_\nu}\right|^\nu\right), \; \lambda_\nu = \left[\frac{2^{(-2/\nu)}\Gamma(1/\nu)}{\Gamma(3/\nu)}\right]^{1/2} \tag{8}$$

where ν is a positive shape parameter governing the thickness of the tail behaviour of the distribution. For $\nu = 1$ GED reduces to the double exponential distribution (Laplace distribution). For $\nu = 2$ GED reduces to the standard normal distribution and for $\nu \to \infty$ to the uniform distribution.

Now that the specification of a GARCH-type model is complete we can estimate the model. Quasi maximum likelihood estimation, method proposed by Bollerslev & Wooldridge (1992) and Berndt-Hall-Hall-Hausman (BHHH) iterative algorithm that is recommended by Bollerslev (1986) provide consistent estimation of the GARCH parameters even when the true density function of the errors is non-Gaussian. This estimation method is built-in EViews 5.1, the package which was used for calculation.

4. Data

The data used in the paper are the daily closing market index MBI-10 from MSE. The Macedonian stock exchange index (MBI-10) - *Makedonski Berzanski Indeks* (in Macedonian) started on 4 January 2005. This index is capitalization-weighted index consisting of up to 10 shares listed on the official market of the MSE at least 20 days before the revision of the index. Shares of individual companies to be included in the MBI-10 index, must satisfy several standard requirements set by the MSE authority. These standard requirements are related to the following: (i) market capitalization (contributing 30% to the MBI-10 index); (ii) daily average turnover of a particular share (20%); (iii) average number of transactions with a particular share (10%); (iv) relative liquidity of the share (20%) and (v) relation between the number of days a particular share was traded and the total number of trading days on the official market (20%).

Before introducing MBI-10 index, another index (MBI) was in use, started on 31 October 2001. However, MBI index was a non-weighted price index, based on five most liquid shares only. Due to the methodological differences between these two indices we decided not to use MBI index. Therefore we based our analysis on the MBI-10 index only.

The data are obtained from the MSE website. The period is from 4/1/2005 to 21/9/2007, with 632 observations. However, 605 observations (4/1/2005 to 14/8/2007) were effectively used to calculate returns summary statistics and for estimation of GARCH models. The last 27 observations were left for examination of the out-of-sample forecasting accuracy.

Throughout this paper, stock market returns are defined as continuously compounded or log returns (hereafter returns) at time t, r_t, calculated as follows:

$$r_t = \log(P_t / P_{t-1}) = \ln P_t - \ln P_{t-1}, \qquad (9)$$

where P_t and P_{t-1} are the closing market index of MBI-10 at days t and $t-1$, respectively.

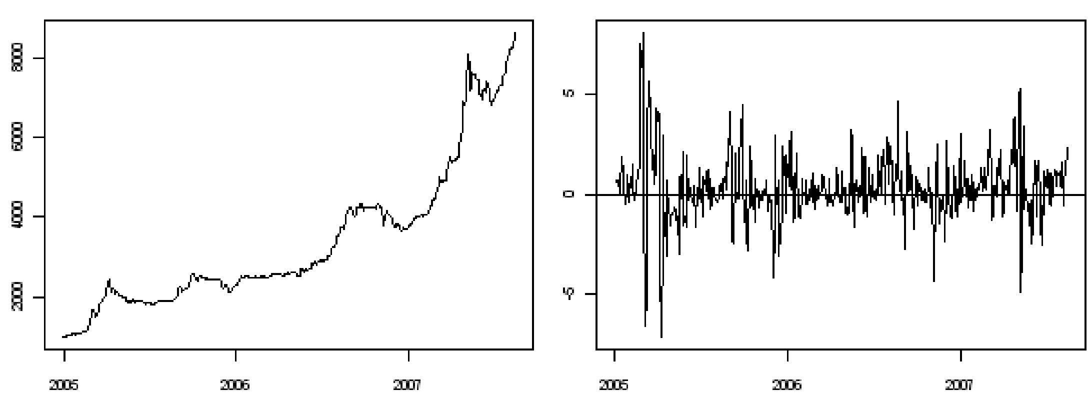
5. Results

5.1. Stylized facts of the MBI-10 returns

The plots of the daily MBI-10 index and returns are given in Figure 1. Visual inspection of MBI-10 returns shows that the mean returns are constant but the variances change over time around some 'normal' level, with large (small) changes tending to be followed by large (small) changes of either sign, i.e. volatility tends to cluster. Periods of high volatility can be distinguished from low volatility periods. It seems that the MBI-10 returns comply with the first and second stylized facts listed in Section 2. Formal tests of GARCH effects for MBI-10 returns are given in the next section, where it is shown that such time-varying effects are indeed evident in the returns series. Therefore it seems appropriate to model MBI-10 returns by using Bollerslev's (1986) GARCH models.

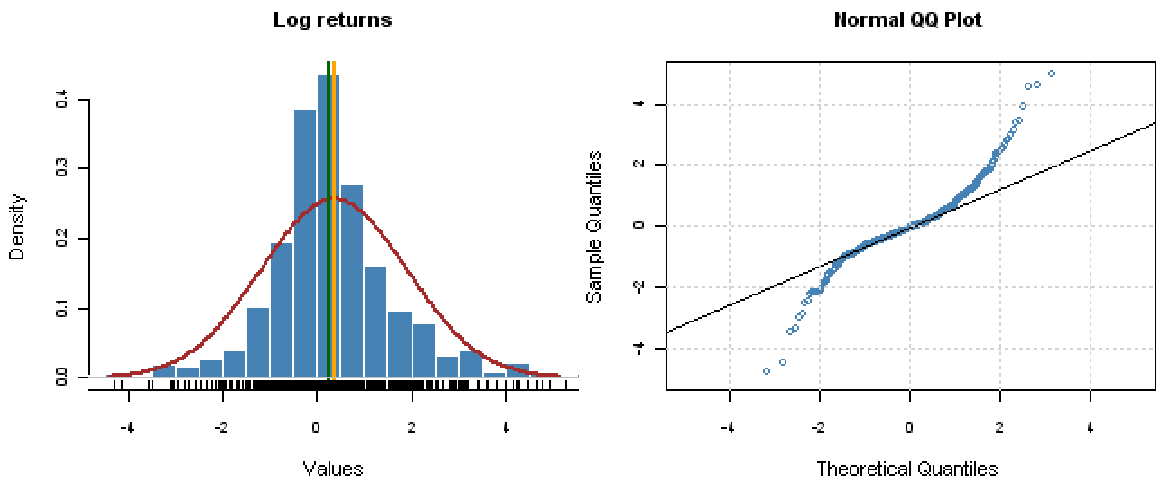
Figure 2 (left) plots a histogram of returns and a Gaussian density whose mean and variance match sample estimates. It shows that numerous returns are above four standard deviations, which is highly unlikely in the Gaussian distribution. The financial time series with such histogram are said to be with heavy tails. The distribution of the MBI-10 returns is characterized not only by heavy tails, but also by a high peakedness at the center, which is the third stylized fact from Section 2.

Figure 1: Daily MBI-10 index and daily returns



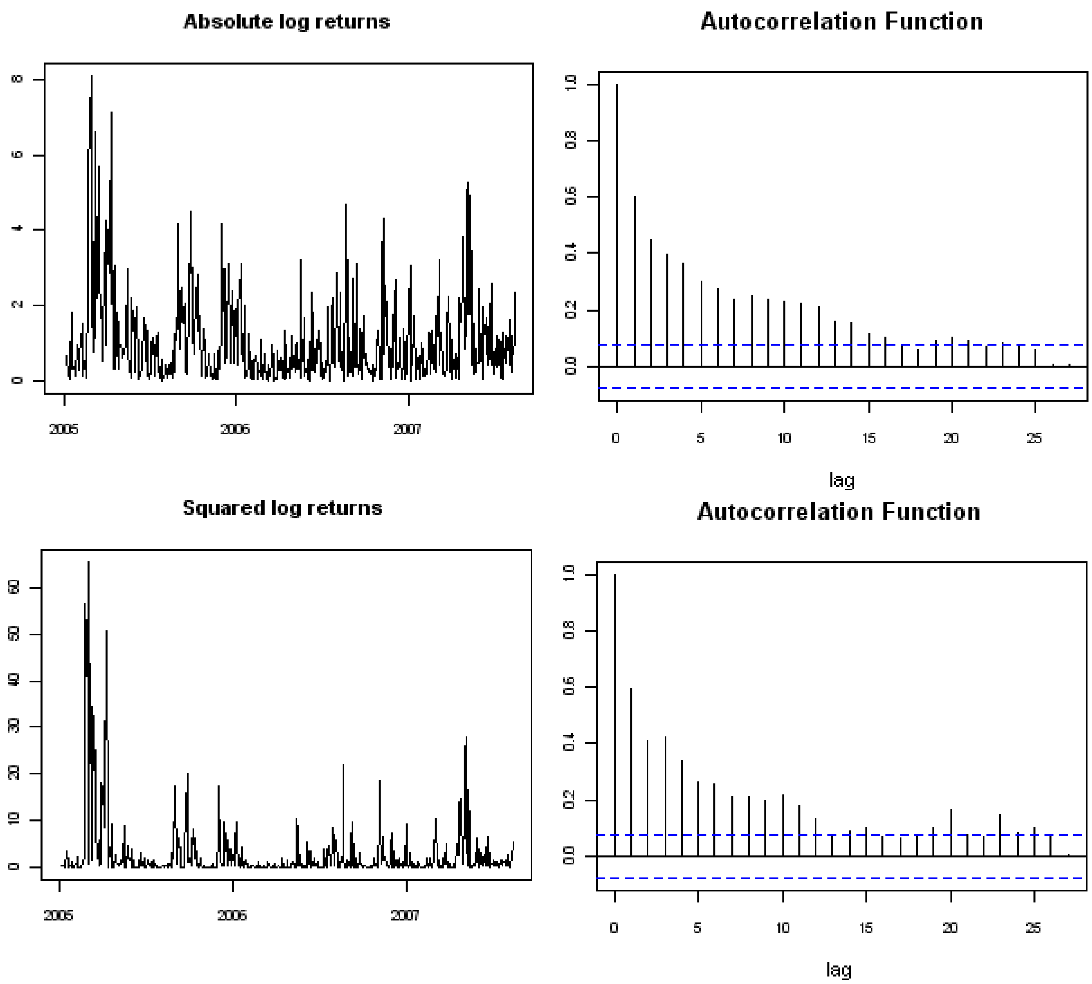
A quantile-quantile (QQ) plot is a graphical tool for checking whether two distributions are of the same type. Since the QQ plot plots quantiles of two distributions, if they are of the same type, the plot should be linear. In this case we are checking whether the empirical distribution of MBI-10 standardized returns and the hypothesized Gaussian distribution are of the same type. The QQ plot in Figure 2 (right) shows clearly that the distribution tails of the MBI-10 are heavier than the tails of the Gaussian distribution. Looking at the tails of this Q-Q plot we can see substantial deviation from the 45 degree line indicating deviation from a Gaussian distribution (the third stylized fact). A detailed description of how to interpret QQ plots in financial applications can be found in Kuczmarski & Rosenbaum (1999).

Figure 2: Normalized return distribution and the Gaussian QQ plot



Since the conditional volatility is not directly observable the absolute or squared returns are used instead. In Figure 3 both absolute and squared MBI-10 returns are shown with their autocorrelation functions. First, both time series plots have “spiky” look demonstrating variation in conditional volatility. When using squared returns, extreme returns contribute more to the conditional volatility, dominating the time series plot. Second, slow decay of autocorrelation in absolute and squared returns is evident from the autocorrelation plots. This is sometimes interpreted as a sign of long-range dependence.

Figure 3: Absolute and squared returns and their autocorrelation functions



5.2. Descriptive statistics and preliminary findings

The main summary statistics and a few tests for the MBI-10 returns are presented in Table 2. The mean daily return of the MBI-10 series is 0.356%. The standard deviation of the daily returns is 1.556% which is equivalent to an annualized volatility of 29.73%. The series also exhibits a positive skewness of 0.47 and an excess kurtosis of 4.75, indicating that the returns are not normally distributed. The Jarque-Bera statistic of 597, much greater than any critical value at conventional confidence levels, thus rejecting the null hypothesis of normally distributed returns. These findings are consistent with previous discussion related to the histogram of returns and QQ plot. The summary statistics and the Jarque-Bera test confirm that the MBI-10 daily returns have thick tails and the non-Gaussian distribution.

Table 2: Summary statistics and diagnostic checks on MBI-10 returns

Mean (in percent)	0.356
Standard deviation (in percent)	1.556
Minimum value (in percent)	-7.130
Maximum value (in percent)	8.090
Skewness	0.470
Excess kurtosis	4.750
Jarque-Bera test statistic	597.0[a]
Number of observations	605
$LB(10)$	301.6[a]
$LB^2(10)$	216.0[a]
Asymmetric GARCH autocorrelation	0.342
Asymmetric GARCH $LB(1)$	71.110[a]
$ADF-$ test (constant, no trend)	-5.520[a]
$ADF-$ test (constant, trend)	-5.510[a]
ERS P_T test (constant, no trend)	0.075[a]
ERS P_T test (constant, trend)	0.259[a]

Note: [a] Denotes significance at the 1% level. Jarque-Bera test statistic tests hypotheses H_0 : returns normally distributed, H_1 : returns not normally distributed. $LB(10)$ and $LB^2(10)$ are Ljung-Box statistics for 10 lags, calculated for returns and squared returns respectively. $LB^2(10)$ statistic tests hypotheses H_0 : volatility clustering, H_1 : no volatility clustering. Ljung-Box statistic for the asymmetric GARCH autocorrelation tests hypotheses H_0 : no leverage effect, H_1 : leverage effect. Augmented Dickey-Fuller (ADF) $t-$ test where the lag lengths in the ADF equations were set to 10. Elliot, Rothenberg and Stock (ERS) point optimal unit root test P_T. Both unit root tests test hypotheses H_0 : unit root, H_1 : no unit root (stationary).

The Ljung-Box statistics $LB(10)$ and $LB^2(10)$ for the returns and squared returns series respectively, are highly significant. Therefore, we reject the hypothesis that there is no autocorrelation in the level of returns and squared returns. The $LB(10)$ test result could be interpreted as an indicator of the Macedonian stock market information inefficiency since there is a strong chance that investors could use historical data to earn above average gains by purchasing and selling stocks. The $LB^2(10)$ test result suggests significant autocorrelation in the squared returns series. In other words, the GARCH effect, i.e. time-varying second moment has been detected in the MBI-10 returns series. Thus the use of GARCH-type models for the conditional variance is justified.

As a very simple test of the leverage effect in the MBI-10 returns series the asymmetric GARCH test was used. This test is a Ljung-Box-type test based on the asymmetric GARCH autocorrelation, which is the first-order autocorrelation coefficient between lagged returns and current squared returns. Since the asymmetric GARCH autocorrelation is equal to 0.342 and highly significant, the hypothesis of no leverage effect has been rejected. This situation is typical for all emerging markets. As the Macedonian stock market is still developing, significant autocorrelation could be caused by nonsynchrounous trading. This is one of the possible explanations, yet another one could be the asymmetric price adjustment. In the next section the existence of the leverage effect is further tested.

The last four statistics in Table 2 are used to test stationarity of the MBI-10 returns series. Stationarity is an important characteristic for time series data. If returns series are found to be nonstationary, it will be necessary to use first differences before we proceed further estimating the GARCH models.

This paper uses the two unit root tests to test the stationary of the MBI-10 return series: ADF test proposed by Said & Dickey (1984) and point optimal unit root test proposed by Elliot, Rothenberg & Stock (1996). Two versions of these tests we applied: with constant and with constant and trend. The test results for the MBI-10 returns series are given in Table 2. The both tests strongly reject the hypothesis of nonstationarity. However, despite the unit root test results that the MBI-10 returns series should be considered stationary, returns display a degree of time dependence, the Ljung-Box statistic for the returns series $LB(10)$ is highly significant.

5.3. Test of asymmetry

To investigate further the existence of leverage effect, the symmetrical GARCH model will be estimated. The joint test for asymmetry as proposed in Engle & Ng (1993) will be conducted on the residuals from a symmetric GARCH(1,1) model. If the symmetric GARCH(1,1) model is a sufficient model for the returns then the residuals from such model will not display any sign bias, negative size bias or positive size bias. Then it would not be justifiable to use an asymmetric conditional volatility model. Otherwise the asymmetric GARCH models described above would be estimated. The specification of the test for asymmetry is as follow:

Sign bias: $e_t^2 = b_0 + b_1 S_{t-1}^- + \nu_t$ (10)

Negative sign bias: $e_t^2 = b_0 + b_1 S_{t-1}^- e_{t-1} + \nu_t$ (11)

Positive sign bias: $e_t^2 = b_0 + b_1 S_{t-1}^+ e_{t-1} + \nu_t$ (12)

Joint test: $e_t^2 = b_0 + b_1 S_{t-1}^- + b_2 S_{t-1}^- e_{t-1} + b_3 S_{t-1}^+ e_{t-1} + \nu_t$ (13)

where S_{t-1}^- is an indicator dummy variable that takes the value 1 if $e_{t-1} < 0$ and zero otherwise and $S_{t-1}^+ = 1 - S_{t-1}^-$.

All t statistics in Table 3 refer to the coefficient b_1 in the first three regressions, while the joint test refers to the coefficients (b_1, b_2, b_3) in the last regression.

Results for the joint test for asymmetry provided in Table 3 show a weak evidence for existence of asymmetry in the MBI-10 returns that remained after estimating the symmetric GARCH(1,1) models. Based on residuals from the GARCH(1,1) models the joint test for asymmetry detected the sign bias and negative size bias effects. However, when the GARCH(1,1)-M models were estimated only the weak evidence for the sign bias effects was found in residuals. These results are robust when the same model was estimated using one of the three error distributions: Gaussian, Student$-t$ and GED distribution. Overall, the joint test provides a weak support for using asymmetric models in the specification of the variance equation for modelling of the MBI-10 returns.

Table 3: Test of asymmetry

Model	**Sign bias $t-$ test**	**Negative size bias $t-$ test**	**Positive size bias $t-$ test**	**Joint test $t-$ test**
GARCH Normal	-1.8571 (0.064)	1.8677 (0.062)	0.6005 (0.548)	1.3101 (0.2701)
GARCH-M Normal	-1.6415 (0.101)	0.5851 (0.559)	0.7257 (0.468)	1.0952 (0.351)
GARCH Student$-t$	-1.7191 (0.086)	2.1038 (0.036)	0.5702 (0.569)	1.1641 (0.323)
GARCH-M Student$-t$	-1.6175 (0.106)	1.1424 (0.254)	0.6271 (0.531)	0.9143 (0.434)
GARCH GED	-1.7535 (0.080)	2.0135 (0.045)	0.5465 (0.585)	1.2020 (0.308)
GARCH-M GED	-1.6882 (0.092)	0.7979 (0.425)	0.6215 (0.535)	1.1056 (0.346)

Note: Numbers in parentheses are the $p-$ values, i.e. marginal significance levels.

5.4. Estimated GARCH models

Before starting the analysis of volatility forecasting models performance, estimated GARCH-type models are discussed. Preliminary investigation identified AR(2)-GARCH(1,1)-M model as an appropriate model to start with. This investigation and lag length selection was based on the Akaike and Schwarz information criteria (*AIC* and *SIC* respectively), significance of the model parameters and the post estimation tests such as Ljung-Box test for model residuals and squared residuals. Table 4 to Table 6 present the estimation results for the mean and variance equations. As it was shown before in Table 2, according to the Ljung-Box test the MBI-10 returns are autocorrelated. The pattern of autocorrelation coefficients of the MBI-10 returns and their significance suggests that they follow an autoregressive process of order 2, i.e. AR(2) process. Therefore the mean equation includes two past return values. These two terms should capture the linear process in the return series. The two AR(2) coefficients are significant at the conventional significance level in all estimated models. Additional term with coefficient λ in the mean equation (1) describes relationship between returns and their volatility. The coefficient λ (risk premium) is significant at the 5% or 10% level in all estimated models, though with the opposite sign than expected. The only exceptions are GARCH(1,1)-M, GJR(1,1)-M and PGARCH(1,1)-M models with assumed Student $-t$ distribution. However, this result is not quite unusual as shown by Glosten, Jagannathan & Runkle (1993). They provided a brief overview of the conflicting results in the literature and then explained why both positive and negative relationship between returns and volatility would be consistent with theory. One of the reasons why the risk premium coefficient is negative could lay in a different reaction of returns on arrival of “bad” and “good news”. That was partially confirmed with our results. All asymmetric models in Table 4 to Table 6 have larger and more significant coefficient λ than the same coefficient in the symmetric GARCH(1,1)-M model.

In the variance equation the first three coefficients: ω (constant), α (ARCH effect) and β (GARCH effect) are highly significant at the conventional significance level and with expected sign. The sizes of the estimated parameters α and β in the GARCH-type models determine the short-run dynamics of the volatility. The sum of estimated α and β is generally less than 1. The only exception are the EGARCH(1,1)-M models where both α and β parameters are overestimated. For other models estimated parameter α belongs to (0.2, 0.3) interval and β to (0.72, 0.75) interval. These values for parameters α and β are consistent with the results obtained for other financial markets (Alexander, 2001). In case of GJR(1,1)-M model with non-Gaussian distributions sum of estimated parameters $\alpha+\beta$ is slightly over 1. However, the regularity condition for the existence of the second moment of GJR(1,1) model is not the same as for GARCH(1,1) model. According to Ling & McAleer (2002b) the regularity condition is $\alpha+\beta+\gamma/2<1$, and it is satisfied for all three estimated GJR(1,1)-M models. Namely, for the GJR(1,1)-M model with Gaussian distribution we have $\alpha+\beta+\gamma/2=0.9318$, for Student $-t$ distribution $\alpha+\beta+\gamma/2=0.9646$ and for GED: $\alpha+\beta+\gamma/2=0.9457$.

Typically for GARCH models for returns data, $\alpha+\beta$ is close to 1, which implies that innovation to the conditional variance will be highly persistent indicating that large changes in returns tend to be followed by large changes and small changes tend to be followed by small changes. This confirms that volatility clustering is observed in the Macedonian MBI-10 index.

The coefficient γ (leverage effect) is significant at the 5% or 10% level in most asymmetric models with assumed non-Gaussian distribution. However, in contrast to the results found for most other markets, the leverage effect term has unexpected negative sign in case of the GJR, TGARCH and PGARCH models and positive in the EGARCH model. The positive innovations would imply a higher next period conditional variance than negative innovations of the same sign, indicating that the existence of leverage effect is not observed in returns of the Macedonian stock market index.

The shape parameters in both non-Gaussian distributions, i.e. degree of freedom in case of Student $-t$ distribution is about 5.5 and GED parameter in case of Generalized Error Distribution is about 1.35 and are highly significant. This justify using non-Gaussian distribution when modeling volatility of the Macedonian returns.

Ljung-Box test was used to check for any remaining autocorrelations in standardized and squared standardized residuals from the estimated variance equation. If the variance equation is specified correctly, two statistics $LB(10)$ and $LB^2(10)$ should not be significant. Indeed, they are not significant at the conventional significance level. Remaining ARCH effects were not been detected in the standardized residuals.

Table 4 to Table 6 clearly show that standardized residuals from all estimated models are not normally distributed. These results are consistent with the findings of other authors (e.g. Poon & Granger, 2003, 2005) that GARCH-type models are not quite successful in capturing the heavy tails in the stock market returns. Models that take into account higher moments and extreme events models would be probably more successful.

Generally, model selection criteria such as AIC and SIC and log likelihood identify GARCH-type models with non-Gaussian distribution as more appropriate for modeling the Macedonian stock market index volatility in comparison to the same class of models, but with Gaussian distribution. Among these models one model clearly stands out, GJR(1,1)-M with Student $-t$ distribution.

To check the robustness of the results obtained, different initial values for the BHHH iterative algorithm were used. The results of these exercises are not presented here, but the BHHH iterative algorithm converged after slightly different number of iteration to the same estimated GARCH models.

Table 4: Estimated GARCH models with Gaussian distribution

Parameter	GARCH	EGARCH	GJR	TGARCH	PGARCH
Mean equation					
ϕ_0 (constant)	0.4273[a] (3.670)	0.4815[a] (4.587)	0.4924[a] (4.187)	0.5290[a] (4.842)	0.4930[a] (4.236)
ϕ_1 (AR(1))	0.5795[a] (13.737)	0.5959[a] (13.193)	0.5941[a] (13.234)	0.6249[a] (12.902)	0.5995[a] (13.090)
ϕ_2 (AR(2))	-0.0805[c] (-1.678)	-0.0817 (-1.599)	-0.0807[c] (-1.673)	-0.0830[c] (-1.641)	-0.0791[c] (-1.617)
λ (risk premium)	-0.2946[c] (-1.873)	-0.3519[b] (-2.359)	-0.3346[b] (-2.021)	-0.4176[a] (-2.747)	-0.3522[b] (-2.155)
Variance equation					
ω (constant)	0.0905[b] (2.522)	-0.2878[a] (-4.780)	0.0939[b] (2.492)	0.1122[a] (3.363)	0.1022[a] (2.756)
α (ARCH effect)	0.2077[a] (3.851)	0.3917[a] (4.877)	0.2434[a] (3.090)	0.1975[a] (4.239)	0.1986[a] (3.540)
β (GARCH effect)	0.7363[a] (12.000)	0.9032[a] (26.365)	0.7444[a] (11.540)	0.7457[a] (13.724)	0.7413[a] (11.760)
γ (leverage effect)		0.0680 (1.018)	-0.1121 (-0.970)	-0.2672 (-1.167)	-0.1826 (-0.933)
δ (power parameter)					1.6131[b] (2.221)
AIC	2.9475	2.9450	2.9435	2.9446	2.9464
SIC	2.9986	3.0034	3.0019	3.0030	3.0121
$-\log L$	881.685	879.914	879.463	879.806	879.343
Skewness	0.1882	0.0034	0.1234	-0.0158	0.0707
Excess kurtosis	5.1182	5.1762	5.2195	5.2814	5.1992
$LB(10)$	9.9445 (0.269)	9.0786 (0.336)	8.6460 (0.373)	7.7846 (0.455)	8.3798 (0.397)
$LB^2(10)$	4.9406 (0.764)	3.8397 (0.871)	5.9165 (0.657)	6.4424 (0.598)	5.9029 (0.658)

Note: [a] Denotes significance at the 1% level, [b] at 5% level, and [c] at 10% level. Numbers in parentheses below coefficient estimates are the Bollerslev-Wooldridge (1992) robust $t-$ statistics. AR(1) and AR(2) denote the own one- and two-period lagged returns, respectively. *AIC*, *SIC* and –log*L* are Akaike information criteria, Schwarz information criteria and negative log likelihood respectively. $LB(10)$ and $LB^2(10)$ are the Ljung-Box statistics for the model standardized and squared standardized residuals using 10 lags, respectively. Numbers in parentheses below the Ljung-Box statistics are the $p-$ values, i.e. marginal significance levels.

Table 5: Estimated GARCH models with Student $-t$ distribution

Parameter	**GARCH**	**EGARCH**	**GJR**	**TGARCH**	**PGARCH**
Mean equation					
ϕ_0 (constant)	0.3478[a] (3.584)	0.3760[a] (4.319)	0.3809[a] (4.011)	0.3879[a] (4.123)	0.3790[a] (4.002)
ϕ_1 (AR(1))	0.5782[a] (13.158)	0.5912[a] (13.564)	0.5868[a] (13.523)	0.5990[a] (14.078)	0.5835[a] (13.462)
ϕ_2 (AR(2))	-0.0894[b] (-2.099)	-0.0821[c] (-1.903)	-0.0895[b] (-2.068)	-0.0810[c] (-1.887)	-0.0903[b] (-2.104)
λ (risk premium)	-0.2090 (-1.495)	-0.2402[c] (-1.861)	-0.2186 (-1.543)	-0.2640[c] (-1.829)	-0.2109 (-1.494)
Variance equation					
ω (constant)	0.0705[a] (2.726)	-0.3059[a] (-5.878)	0.0732[a] (2.884)	0.1044[a] (3.333)	0.0652[a] (2.714)
α (ARCH effect)	0.2377[a] (4.003)	0.4143[a] (5.679)	0.2946[a] (3.759)	0.2344[a] (4.887)	0.2041[a] (3.252)
β (GARCH effect)	0.7372[a] (15.031)	0.9204[a] (37.430)	0.7391[a] (15.609)	0.7279[a] (14.488)	0.7391[a] (14.216)
γ (leverage effect)		0.0756[c] (1.799)	-0.1383[c] (-1.710)	-0.2657[b] (-2.303)	-0.1327 (-1.495)
δ (power parameter)					2.3585[a] (2.928)
t degree of freedom	5.5202[a] (4.456)	5.5617[a] (4.269)	5.5577[a] (4.423)	5.5515[a] (4.242)	5.5739[a] (2.928)
AIC	2.8823	2.8852	2.8801	2.8854	2.8830
SIC	2.9407	2.9509	2.9458	2.9511	2.9560
$-\log L$	861.004	860.877	859.353	860.958	859.231
Skewness	0.2071	0.0223	0.1115	-0.0425	0.1602
Excess kurtosis	5.3346	5.3337	5.4725	5.3559	5.5147
$LB(10)$	10.4250 (0.236)	8.2186 (0.412)	9.1289 (0.332)	8.0817 (0.426)	9.6196 (0.293)
$LB^2(10)$	4.8844 (0.770)	4.1718 (0.841)	5.4341 (0.710)	5.4912 (0.704)	5.3821 (0.716)

Note: [a] Denotes significance at the 1% level, [b] at 5% level, and [c] at 10% level. Numbers in parentheses below coefficient estimates are the Bollerslev-Wooldridge (1992) robust $t-$ statistics. AR(1) and AR(2) denote the own one- and two-period lagged returns, respectively. *AIC*, *SIC* and $-\log L$ are Akaike information criteria, Schwarz information criteria and negative log likelihood respectively. $LB(10)$ and $LB^2(10)$ are the Ljung-Box statistics for the model standardized and squared standardized residuals using 10 lags, respectively. Numbers in parentheses below the Ljung-Box statistics are the $p-$ values, i.e. marginal significance levels.

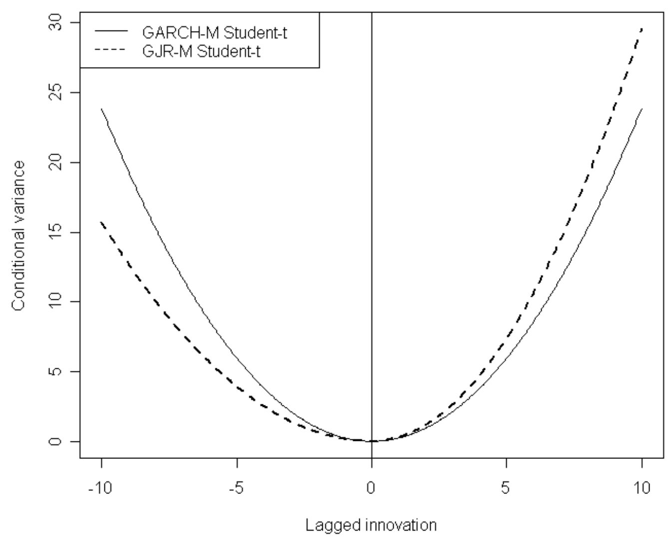
Table 6: Estimated GARCH models with GED distribution

Parameter	GARCH	EGARCH	GJR	TGARCH	PGARCH
Mean equation					
ϕ_0 (constant)	0.3748[a] (3.705)	0.4268[a] (4.567)	0.4143[a] (4.148)	0.4497[a] (4.500)	0.4149[a] (4.148)
ϕ_1 (AR(1))	0.5885[a] (13.510)	0.6085[a] (14.203)	0.6009[a] (14.056)	0.6200[a] (15.022)	0.6016[a] (14.057)
ϕ_2 (AR(2))	-0.0939[b] (-2.230)	-0.0925[b] (-2.208)	-0.0927[b] (-2.186)	-0.0953[b] (-2.305)	-0.0927[b] (-2.178)
λ (risk premium)	-0.2542[c] (-1.702)	-0.3293[b] (-2.352)	-0.2829[c] (-1.848)	-0.3686[b] (-2.371)	-0.2847[c] (-1.852)
Variance equation					
ω (constant)	0.0779[a] (2.841)	-0.2999[a] (-5.795)	0.0794[a] (3.065)	0.1060[a] (3.391)	0.0806[a] (3.054)
α (ARCH effect)	0.2203[a] (4.086)	0.4009[a] (5.611)	0.2598[a] (3.945)	0.2059[a] (4.984)	0.1967[a] (3.851)
β (GARCH effect)	0.7371[a] (14.855)	0.9079[a] (34.270)	0.7462[a] (16.316)	0.7429[a] (15.380)	0.7457[a] (15.199)
γ (leverage effect)		0.0728[c] (1.853)	-0.1206[c] (-1.774)	-0.2699[b] (-2.400)	-0.1589[c] (-1.749)
δ (power parameter)					1.9568[a] (3.092)
GED parameter	1.3481[a] (13.929)	1.3548[a] (13.684)	1.3550[a] (13.803)	1.3553[a] (13.559)	1.3553[a] (13.672)
AIC	2.8983	2.8985	2.8962	2.8983	2.8994
SIC	2.9567	2.9642	2.9619	2.9640	2.9724
$-\log L$	865.822	864.894	864.190	864.848	864.166
Skewness	0.1922	-0.0072	0.1129	-0.0354	0.1066
Excess kurtosis	5.2405	5.2721	5.3668	5.3646	5.3625
$LB(10)$	9.6914 (0.287)	7.9660 (0.437)	8.1703 (0.417)	7.8623 (0.447)	8.1175 (0.422)
$LB^2(10)$	4.8643 (0.772)	3.8380 (0.871)	5.7217 (0.678)	6.3101 (0.613)	5.7259 (0.678)

Note: [a] Denotes significance at the 1% level, [b] at 5% level, and [c] at 10% level. Numbers in parentheses below coefficient estimates are the Bollerslev-Wooldridge (1992) robust $t-$ statistics. AR(1) and AR(2) denote the own one- and two-period lagged returns, respectively. *AIC*, *SIC* and –log*L* are Akaike information criteria, Schwarz information criteria and negative log likelihood respectively. $LB(10)$ and $LB^2(10)$ are the Ljung-Box statistics for the model standardized and squared standardized residuals using 10 lags, respectively. Numbers in parentheses below the Ljung-Box statistics are the $p-$ values, i.e. marginal significance levels.

News impact curve introduced by Pagan & Schwert (1990) provides graphical representation of the degree of asymmetry of volatility. The news impact curve is drawn by using estimated variance equation and successive values of innovations to find out what the corresponding values of conditional variance derived from the model would be. The two curves on Figure 4 are drawn by using the estimated variance equation for GARCH(1,1)-M and GJR(1,1)-M models assuming Student $-t$ distribution. As can be seen from Figure 4 the GARCH(1,1)-M news impact curve is symmetric about zero. The other news impact curve is asymmetric with positive innovations having more impact on future volatility than negative of the same magnitude. As discussed before, this is a bit unusual result, since a positive sign of the γ coefficient (leverage effect) was expected. However, the level of asymmetry is not high, i.e. these two curves are moving close to each other.

Figure 4: News impact curves for GARCH-M and GJR-M models



5.5. Forecasting accuracy

To see how the models fit past data in-sample forecasts have been generated. Although the paper focuses on the conditional variance and not on the returns themselves, for illustrative purposes, Figure 5 shows MBI-10 returns and in-sample forecasts based on the GJR(1,1)-M model with assumed Student $-t$ distribution, i. e. the mean equation of the GJR(1,1)-M model. Variation in the daily MBI-10 return series are captured well. However, the extreme values in MBI-10 returns are not reproduced quite accurately.

Figure 5: In-sample returns forecasts with GJR-M Student $-t$ model

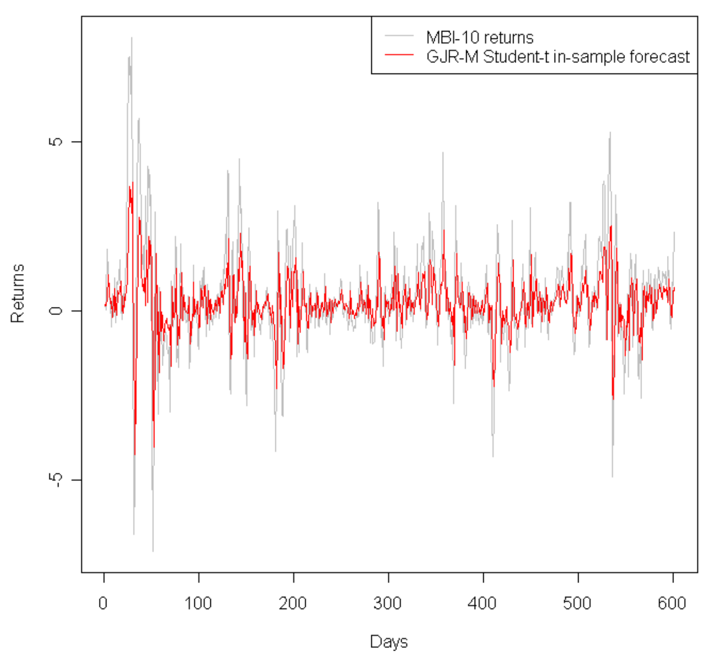
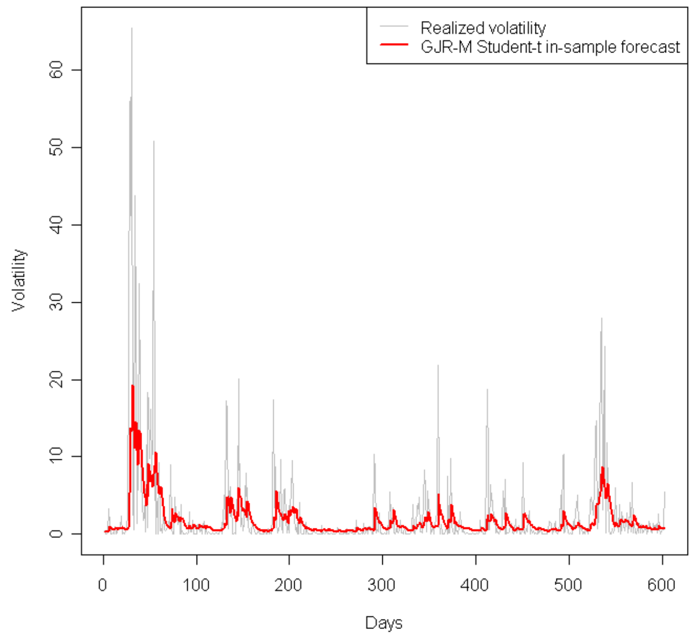


Figure 6 shows the behavior of the realized volatility and in-sample static forecast based on the GJR(1,1)-M model with assumed Student $-t$ distribution. Since the actual volatility is unobserved the different estimators were used in empirical studies, usually based on the higher frequency data with intra-day intervals or daily high/low returns. However, for MBI-10 index only the closing values were available and therefore the squared return series was used as a proxy for the realized volatility. The graph provides an indication on the GJR(1,1)-M model ability to track variation in realized volatility. Obviously the largest spikes in the realized volatility are not captured well. To model these peaks in the realized volatility above some high threshold the extreme values modeling approach would be probably more appropriate.

Figure 6: In-sample volatility forecasts with GJR-M Student $-t$ model



The forecasting performance of each model is evaluated both in-sample and out-of-sample by using three symmetric and two asymmetric measures. Three standard symmetric measures, i.e. loss functions used to evaluate in-sample and out-of-sample forecasting accuracy are: the root mean square error ($RMSE$), the mean absolute error (MAE) and the Theil inequality coefficient (TIC). The $RMSE$ is defined by

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}, \tag{14}$$

where $\hat{\sigma}_t^2$ is the one-step-ahead volatility forecast, σ_t^2 is the actual volatility and T is a number of forecasts. The MAE is defined by:

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| \tag{15}$$

The TIC is defined by:

$$TIC = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2)^2}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma_t^2)^2}} \tag{16}$$

The Theil inequality coefficient is the scaled measure that always lies between zero and one, where zero indicates a perfect fit.

Two asymmetric measures proposed by Brailsford & Faff (1996) are based on the mean mixed error (MME) statistics and are defined as:

$$MME(U) = \frac{1}{T}\left[\sum_{t=1}^{O}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| + \sum_{t=1}^{U}\sqrt{\left|\hat{\sigma}_t^2 - \sigma_t^2\right|}\right] \tag{17}$$

$$MME(O) = \frac{1}{T}\left[\sum_{t=1}^{O}\sqrt{\left|\hat{\sigma}_t^2 - \sigma_t^2\right|} + \sum_{t=1}^{U}\left|\hat{\sigma}_t^2 - \sigma_t^2\right|\right] \tag{18}$$

where O and U is the number of over ($\hat{\sigma}_t^2 > \sigma_t^2$) and under prediction ($\hat{\sigma}_t^2 < \sigma_t^2$) respectively. $MME(O)$ penalizes more heavily the over predictions and $MME(U)$ penalizes more heavily the under predictions. The main reason for introducing asymmetric measures is that investors do not give equal importance to over- and under-prediction of volatility. For example, in the pricing of options, while over-prediction is undesirable for buyers, under-prediction is undesirable for sellers.

Table 7 reports the value and ranking of all fifteen competing models under *RMSE, MAE, TIC, MME(U)* and *MME(O)* for in-sample of the MBI-10 volatility forecasts. Similarly Table 8 reports value of the five forecasting accuracy criteria and ranking of all fifteen competing models for out-of-sample of the MBI-10 volatility forecasts. According to the forecasting accuracy criteria used, there is consistency to choose among the models in case of in-sample forecasting. Within the GARCH-type models, the ranking of any forecasting model varies depending upon the choice of error distribution. Under the three distributions, the performance of GARCH and EGARCH models is not as good as that of GJR and PGARCH models. The Student $-t$ distribution seems a little more accurate than the other two distributions. In Table 7 for in-sample forecasts, asymmetric models with non-Gaussian distributions, the Student $-t$ distribution in particular, are ranked higher than the other estimated models. The GJR(1,1)-M model with the Student $-t$ distribution is the highest ranked model according to *RMSE, TIC* and *MME(U)* criteria. However, it is difficult to choose between this and other models considered. Note that the maximum superior performance of GJR(1,1)-M model compared to other models according to the three symmetric criteria is between 1.8% (*MAE*) and 8.9% (*TIC*) only.

Table 7: Evaluation of the in-sample volatility forecasts

Model	*RMSE*			*MAE*			*TIC*			*MME(O)*			*MME(U)*		
	Actual	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**
GARCH Normal	5.814	0.998	14	2.311	0.982	3	0.607	0.985	13	1.267	0.930	1	2.225	0.995	13
EGARCH Normal	5.826	1.000	15	2.325	0.988	13	0.617	1.000	15	1.278	0.938	3	2.236	1.000	15
GJR Normal	5.702	0.979	7	2.321	0.986	9	0.582	0.944	9	1.316	0.966	7	2.201	0.984	9
TGARCH Normal	5.740	0.985	9	2.353	1.000	15	0.581	0.942	8	1.362	1.000	15	2.201	0.985	10
PGARCH Normal	5.702	0.979	6	2.321	0.986	11	0.580	0.941	7	1.322	0.970	10	2.195	0.982	7
GARCH Student	5.749	0.987	10	2.315	0.984	7	0.588	0.953	10	1.300	0.954	6	2.199	0.983	8
EGARCH Student	5.769	0.990	11	2.322	0.987	12	0.603	0.977	12	1.296	0.951	5	2.216	0.991	12
GJR Student	5.617	0.964	1	2.311	0.982	2	0.562	0.911	1	1.337	0.981	12	2.164	0.968	1
TGARCH Student	5.664	0.972	3	2.317	0.985	8	0.568	0.922	3	1.340	0.984	13	2.169	0.970	3
PGARCH Student	5.623	0.965	2	2.314	0.983	5	0.564	0.914	2	1.336	0.980	11	2.168	0.970	2
GARCH GED	5.791	0.994	12	2.308	0.981	1	0.601	0.974	11	1.273	0.935	2	2.213	0.990	11
EGARCH GED	5.805	0.996	13	2.321	0.986	10	0.612	0.992	14	1.280	0.940	4	2.227	0.996	14
GJR GED	5.678	0.975	5	2.314	0.983	6	0.576	0.934	5	1.318	0.968	8	2.187	0.978	5
TGARCH GED	5.718	0.982	8	2.335	0.992	14	0.578	0.937	6	1.345	0.988	14	2.188	0.979	6
PGARCH GED	5.676	0.974	4	2.314	0.983	4	0.576	0.933	4	1.319	0.968	9	2.186	0.978	4

Note: Actual is the calculated measure. Relative is the ratio between the actual measure of a model and that of the worst performing model. The best performing model has a rank 1.

Table 8: Evaluation of the out-of-sample volatility forecasts

Model	*RMSE*			*MAE*			*TIC*			*MME(O)*			*MME(U)*		
	Actual	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**	**Actual**	**Relative**	**Rank**
GARCH Normal	4.012	1.000	15	2.190	0.982	9	0.569	1.000	15	1.260	0.898	4	2.163	0.991	13
EGARCH Normal	3.858	0.961	5	2.172	0.974	5	0.532	0.935	6	1.305	0.930	9	2.120	0.971	6
GJR Normal	3.922	0.978	12	2.143	0.962	1	0.548	0.963	12	1.252	0.893	3	2.096	0.960	1
TGARCH Normal	3.822	0.952	2	2.223	0.997	13	0.514	0.902	1	1.403	1.000	15	2.116	0.969	5
PGARCH Normal	3.889	0.969	7	2.164	0.971	4	0.538	0.945	8	1.289	0.919	7	2.114	0.969	4
GARCH Student	4.001	0.997	13	2.225	0.998	14	0.557	0.978	13	1.311	0.934	11	2.182	1.000	15
EGARCH Student	3.870	0.965	6	2.191	0.983	10	0.531	0.933	5	1.321	0.942	12	2.138	0.980	11
GJR Student	3.896	0.971	8	2.177	0.977	8	0.534	0.939	7	1.292	0.921	8	2.137	0.979	10
TGARCH Student	3.811	0.950	1	2.229	1.000	15	0.514	0.903	2	1.390	0.991	14	2.146	0.983	12
PGARCH Student	3.914	0.975	10	2.174	0.975	7	0.539	0.947	9	1.286	0.917	6	2.132	0.977	9
GARCH GED	4.011	1.000	14	2.200	0.987	11	0.566	0.995	14	1.269	0.905	5	2.171	0.995	14
EGARCH GED	3.852	0.960	4	2.173	0.975	6	0.531	0.933	4	1.309	0.933	10	2.122	0.972	7
GJR GED	3.917	0.976	11	2.147	0.963	2	0.545	0.957	11	1.248	0.889	1	2.111	0.967	2
TGARCH GED	3.823	0.953	3	2.217	0.995	12	0.517	0.908	3	1.384	0.986	13	2.129	0.975	8
PGARCH GED	3.913	0.975	9	2.149	0.964	3	0.544	0.956	10	1.251	0.891	2	2.111	0.967	3

Note: Actual is the calculated measure. Relative is the ratio between the actual measure of a model and that of the worst performing model. The best performing model has a rank 1.

Generally, the relative differences between forecasting performances of the GARCH-type models are quite small. The largest relative differences between the best and the worst models, based on *TIC* criteria, are 8.9% (in-sample) and 9.8% (out-of-sample) respectively.

When comparing models based on asymmetric accuracy criteria, *MME(U)* and *MME(O)*, the results differ significantly. While *MME(U)* criteria, which penalizes under-prediction, gives ranking that matches ranking based on *RMSE*, *MAE* and *TIC* criteria, *MME(O)* gives almost inverse ranking with symmetric GARCH and EGARCH models performing better than other models considered.

In case of out-of-sample forecasts (Table 8), the performance of these models tends to be rather mixed and quite different from the in-sample results. The only clear pattern in Table 8 or conclusion which can be drawn is that of superiority of asymmetric GARCH models: TGARCH model favoured by *RMSE* and *TIC* and GJR model favoured by *MAE*, *MME(O)* and *MME(U)* criteria. It should be noted that out-of-sample evaluation was based on a rather small sample and therefore results in Table 8 should be taken cautiously and not as a definite answer about forecasting performance of these models.

6. Conclusion

Stock prices volatility is an important factor in portfolio selection, asset pricing, value-at-risk and option pricing where it is used as a measure of risk. The practical aspect of the risk management and the development in econometric modeling of conditional variance, ARCH-type models in particular, attracted attention both academics and practitioners in the last two decades to the problems of modeling and volatility forecasting.

We used a stock market index from Macedonia, a country not previously considered in the volatility literature, to answer four questions raised in the Introduction section. Based on the results presented, the following can be concluded:

1. The stylized facts listed in the Section 2.1 were also identified in the MBI-10 returns by using formal statistical tests and graphs of the MBI-10 returns, corresponding functions and estimated GARCH-type models. Typically for estimated GARCH-type models based on the returns data, the sum of the ARCH and GARCH coefficients is close to unity. This implies that innovations in the conditional variance will be highly persistent indicating that large changes in returns tend to be followed by large changes and small changes tend to be followed by small changes, which means that volatility clustering is observed in the Macedonian financial returns series.
2. To address the question about the impact conditional variance might have on stock returns several univariate GARCH-in-mean-type models were specified: a symmetric GARCH model and four asymmetric models (EGARCH, GJR, TGARCH and PGARCH). The parameter describing the conditional variance, i.e. conditional standard deviation in the mean equation, measuring the risk premium effect, is statistically weakly significant across all models. However, the sign of the risk premium parameter is negative. The implication is that increase in volatility would decrease returns, which is an unexpected result, but could be theoretically justified. Engle & Ng (1993) test of asymmetry provided a weak evidence of asymmetric behavior of the conditional variance. To explore this further and see whether this asymmetric behavior could be attributed to the leverage effect a set of asymmetric GARCH-type models were considered. Estimated models in Table 4 – Table 6 show weakly significant leverage effect parameter only in case of non-Gaussian distributions. The implication of the negative sign in case of the leverage effect parameter is that “bad news” would decrease volatility, while the “good news” would increase volatility indicating that the existence of leverage effect is not observed in the Macedonian returns. These two rather unusual results related to the risk premium and leverage effects, i.e. anomalies in stock market behavior could be expected in the early period of emerging stock markets such as the Macedonian stock market.

3. The estimated models in Table 4 – Table 6 clearly show that the results related to the relationship between returns and conditional volatility can be regarded as quite robust across the models and alternative error distributions.
4. According to the in-sample statistics and out-of-sample forecasts the results in Table 7 and Table 8 indicate, that the forecasting performance of asymmetric GARCH models (GJR and TGARCH in particular) is better than symmetric GARCH models, but with little gain. The models with heavy-tailed asymmetric distributions such as the Student $-t$ distribution rank better than models with other distributions, but again the difference is small. Depending on the accuracy criteria used, the relative differences are between minimum of 2% (*MAE* criteria in case of the in-sample forecasts) to maximum of 9.8% (*TIC* criteria in case of out-of-sample forecasts). Although we cannot find one model that performs best under all the criteria, we can argue that the AR(2)-GJR(1,1)-M model coupled with a Student $-t$ distribution performs very well with the MBI-10 returns.

This study is subject to certain reservations. At the same time these reservations outline directions for future researches that could be investigated to improve the modeling and volatility forecasts of the Macedonian stock market returns. First, the time series of returns is quite short. Longer time series would allow estimation with greater precision, estimation of the GARCH-type models for sub-periods or using of the "rolling windows". That would check the stability of estimated relationship between returns and volatility and how it evolves through time. Second, only symmetric Gaussian and non-Gaussian distributions were used. Assuming an asymmetric non-Gaussian error distribution, such as an asymmetric Student $-t$ or GED distributions, would increase flexibility in modeling of the conditional variance. Third, squared returns were used as a proxy for the realized volatility. The "true volatility" could be better estimated by selecting shorter time intervals, i.e. by using intra-day trading data or minimal and maximal values of returns when such data become available.

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Appendix

Appendix: Summary of the volatility research (countries of former Yugoslavia)

Author	**Country (Index)**	**Data period & frequency**	**Method/Model used**	**Main findings**
Žiković (2007)	Slovenia (SBI-20)	1-Jan-00 – 31-Dec-05; daily	ARMA-GARCH and bootstrapping,	Semi-parametric approach to forecasting VaR was developed. It was confirmed that common VaR models that are widely used in mature markets, such as historical simulation, variance-covariance model and RiskMetrics system are not well suited to transitional capital markets.
Anatolyev (2006)	Croatia (CROBEX), Slovenia (SBI)	Jan-97 – Jan-05; weekly	Nonparametric retrospective and monitoring tests	For analysis of predictability of stock market indexes two nonparametric test were constructed. In case of Slovenia neither retrospective nor monitoring tests detected mean predictability. In case of Croatia retrospective tests strongly reject conditional mean independence.
Miljković & Radović (2006)	Serbia (BELEX-15, BELEXfm, A2007)	4-Oct-05 – 20-Nov-06 (BELEX-15) 1-Sep-05 – 20-Nov-06 (BELEXfm) 10-Jan-05 – 20-Nov-06 (A2007)	Descriptive statistics and ARCH-type of tests	Paper illustrates some of the stylized facts identified in the financial time series. Three stock indices from the Belgrade stock exchange were used for illustration.
Onay (2006).	Croatia (CROBEX)	27-Oct-00 – 26-Aug-05; weekly	Engle and Granger and Johansen cointegration tests; Granger causality test	While the results of Johansen test suggest non-cointegration, Engle-Granger tests reveal a causal flow from European indices to Croatian index.
Posedel (2006)	Croatia (local currency vs. Euro)	2-Jan-01 – 30-Dec-05; daily	Nonlinear-in-mean asymmetric GARCH	NGARCH model was used for option pricing. This model better describes short-run dynamics of the currency series.
Samitas, Kenourgios & Paltalidis (2006)	Croatia (CROBEX), Serbia (BELEX), Macedonia (MBI-10)	Jan-00 – Apr-06; daily	Markov switching regime regression	Possible linkages between the Balkans and developed markets were tested. The Balkans markets display equilibrium relations with their mature counterparts (US, UK, & Germany), supporting the hypothesis that there are interdependencies between emerging and developed stock markets.
Syllignakis & Kouretas (2006)	Slovenia (SBI)	1-Jan-95 – 25-Dec-05; daily and weekly	Markov switching ARCH-L, dynamic conditional	DCC-GARCH(1,1) model reveals a sharp decline in the intensity of the co-movements between Slovenia and Germany stock market after the Russian crisis. Markov switching ARCH-L

			correlation DCC-GARCH	model was used to study for structural breaks in volatility. It is revealed that the conditional volatility has increased over 200% during the Russian crisis.
Žiković (2006a).	Croatia (CROBEX, VIN)	4-Jan-00 – 4-Jan-06	VaR methodology, hybrid approach and historical simulation	Kupiec test and out-of-sample forecasting accuracy have been evaluated for two Croatian stock market indexes. Hybrid approach outperformed historical simulation models.
Žiković (2006b)	Croatia (CROBEX)	7-Apr-03 – 7-Apr-05; daily	VaR methodology and historical simulation	Acceptance of measuring VaR with historical simulation in Croatian financial market was tested. Only models were the historical simulation using 50 and 175 days observation period demonstrated good performance.
Hasan & Quayes (2005)	Slovenia(SBI)	95 - 02; weekly	Standard correlation coefficients and Johansen's cointegration tests	The objective of the study was to estimate the level of integration between the financial markets in nine transitional economies of Europe, Russia and that of the United States. It was shown that none of these markets are either correlated or have any long run relationship with the financial markets in the US. Furthermore, Slovenia does not have any long-term relationship with any of the other nine.
Levaj, Kamenarić, Mišković & Mokrovčak (2005)	Croatia (stock prices for Podravka company)	2-Jan-01 – 10-May-05; daily	GARCH(1,1)	Estimation of the GARCH(1,1) model for the company's stock data was used to illustrate use of the GARCH-type models in forecasting volatility.
Ajayi, Mehdian & Perry (2004)	Croatia (CROBEX), Slovenia (SBI-20)	20-Jul-99 - 6-Sep-02 (Croatia), 1-Sep-94 - 6-Sep-02 (Slovenia); daily	OLS regression with daily dummy variables	There are statistically significant day-of-the-week effects in the stock returns in the case of Slovenia, which has a negative Tuesday effect and positive Thursday and Friday effects.
Égert & Koubaa (2004)	Slovenia (SBI)	2-Jan-94 – 2-Jul-02; daily	GARCH, QGARCH, LSTGARCH, GJR, ESTGARCH	In case of GARCH model for Slovenia $\alpha + \beta > 1$ was obtained. Other tests also identified inadequacy of GARCH model for Slovene index. GJR and QGARCH models reasonably well modeled SBI index.
Fruk (2004)	Croatia (CROBEX)	Apr-97 – Mar-04; monthly	Hylleberg, Engle, Granger & Yoo seasonality test	Hylleberg, Engle, Granger & Yoo seasonality test was applied to the stock returns. Hypothesis of seasonal unit root in CROBEX was rejected.
Tonchev & Kim (2004)	Slovenia (SBI-20, SBI-20NT)	4-Jul-00 – 18-Jun-03; daily	The OLS regression with daily dummy variables and GARCH models	The calendar effects in mean stock returns studied by the OLS regression with dummy variables identified weak evidence for the day of the week effect in mean in Slovenia, but in the opposite direction (reverse effects in positive returns). On the other hand, GARCH models with dummies were employed for testing for calendar effects in the

				conditional variance of returns. They identified the January effect for Slovenia, some weak evidence for monthly seasonality in variance and the reverse half-month effect.
Latković (2002)	Croatia (CROBEX)	1-Jan-97 – 1-Oct-01; daily	GARCH(1,1)	The main issues and methodology of the risk management are discussed. GARCH(1,1) model was used to illustrate risk calculation on the Croatian capital market.
Latković (2001)	Croatia (CROBEX and 12 different companies indices)	1-Sep-97 – 30-Dec-00; daily	CAPM model	CAPM model was used as a framework for analysis and calculating betas.
Deželan (2000)	Slovenia (SBI and LB13)	3-Jan-94 – 5-Mar-98; daily	Runs test, variance ratio test and market model	The hypothesis of a weak form of efficiency of the Slovenian stock market was rejected.
Šestović & Latković (1998)	Croatia (CROBEX, PLI-AA, ZAB-O)	3-Sep-96 – 31-Dec-97; daily	GARCH(1,1)	For the Zagreb stock exchange index CROBEX, estimated GARCH(1,1) model gives $\alpha+\beta$ close to 1. For the Pliva Company index (PLI-AA) and the Zagreb Bank index (ZAB-O) $\alpha+\beta$ is well below 1.